Math 1131
Autumn 2015
Final Exam
Form A

Name: ______________________

Name.nn: ______________________

Lecturer: ______________________

Rec. Instructor: ______________________

Rec. Time: ______________________

Instructions:

- You have **1 hour and 45 minutes** to complete this exam. It consists of 10 questions on 13 pages including this cover sheet and is worth a total of 200 points. The value of each question is listed below and with each question. Partial credit might not be awarded on some questions.
- You may not use any books or notes during this exam.
- Calculators are permitted EXCEPT those calculators that have symbolic algebra or calculus capabilities. In particular, the following calculators and their upgrades are not permitted: TI-89, TI-92, and HP-49. In addition, neither PDAs, laptops nor cell phones are permitted.
- Make sure to read each question carefully.
- Please **write clearly** and make sure to **justify your answers**. Correct answers with no supporting work may receive no credit. Unless otherwise stated, solutions found by graphing will receive no credit.
- Unless otherwise specified, make sure your answers are in **exact form** (i.e. not decimal approximations).
- Please write your answers on the indicated lines.

Question	Point Value	Score	Question	Point Value	Score
1	27		6	28	
2	19		7	19	
3	15		8	17	
4	32		9	8	
5	15		10	20	
			Total	200	

(1). (27 points) Given the function

$$f(x) = \begin{cases} \dfrac{(x+2)(x+5)(x-5)}{13(x+3)(x+5)} & \text{if } x < 2 \\[2ex] \dfrac{4(4-x)(x-5)}{13(x-1)(x+8)} & \text{if } x \geq 2 \end{cases}$$

Find the following:

(a) (3 points) $\lim_{x \to 1} f(x) =$ ____________

(b) (3 points) $\lim_{x \to -\infty} f(x) =$ ____________

(c) (3 points) $\lim_{x \to 2^-} f(x) =$ ____________

(d) (3 points) $\lim_{x \to -3^-} f(x) =$ ____________

(e) (3 points) $\lim_{x \to 2^+} f(x) =$ ____________

(f) (3 points) $\lim_{x \to \infty} f(x) =$ ____________

(g) (3 points) $\lim_{x \to 2} f(x) =$ ____________

(h) (3 points) $\lim_{x \to -5} f(x) =$ ____________

(i) (3 points) Find all values of x such that $f(x)$ is not continuous. ____________

(2). (19 points) Given the function

$$f(x) = 5e^{32-2x^2}$$

(a) (6 points) Use any of the techniques for differentiation to find the derivative, $f'(x)$ of this function.

(b) (6 points) Use any of the techniques for differentiation to find the second derivative, $f''(x)$ of this function.

(c) (4 points) Find the slope of the line tangent to the graph of $f(x)$ when $x = 4$.

(d) (3 points) Find an equation of the line tangent to the graph of $f(x)$ when $x = 4$.

(3). (15 points) Use the definition of the derivative to find $f'(x)$ where $f(x) = \dfrac{3}{2x-5}$

$$\left(\text{Hint: Recall that the definition of the derivative is } f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}\right)$$

Answer (3): $f'(x) =$ ________________________

(4). (32 points) Use any of the techniques for differentiation to find the derivative, $\frac{dy}{dx}$, of each of the following functions: (You do not need to simplify your answers.)

(a) (6 points) $y = \dfrac{2x^3 - 4}{\sqrt[5]{5x^2 - 3}}$

Answer (4a): $\dfrac{dy}{dx} =$ ________________________

(b) (6 points) $y = e^{2x-1}(x^{3/5} + 8)$

Answer (4b): $\dfrac{dy}{dx} =$ ________________________

(Problem (5) cont.)

(c) (10 points) $y = \ln\left[\dfrac{(x+2)^4}{(x-5)^3(x+3)^7}\right]$ *(Hint: simplify first)*

Answer (4c): $\dfrac{dy}{dx} =$ ______________________

(d) (10 points) $y = (x+2)^{x^3-5}$

Answer (4d): $\dfrac{dy}{dx} =$ ______________________

(5). (15 points) The demand equation for a monopolist's product is:

$$p = 500 - 3q$$

and the total-cost function is:

$$c = 0.3q^2 + 38q + 160$$

Find the profit-maximizing output and profit-maximizing price.

Answer (5): Profit-maximizing output = ______________________

Profit-maximizing price = ______________________

(6). (28 points) The following information is given about the function $f(x)$:

- $f(x)$ is continuous at all x except $x = -4$ and $x = 3$.
- $f(-7) = 5,\ f(0) = 1,\ f(8) = 3$.
- $f(x)$ has a vertical asymptote at $x = -4$ and at $x = 3$.
- $\lim_{x \to \infty} f(x) = 6$.
- $f'(x) > 0$ on the intervals $(-\infty, -4), (-4, -1), (1, 3)$ and $(6, \infty)$.
- $f'(x) < 0$ on the intervals $(-1, 1)$ and $(3, 6)$.
- $f''(x) > 0$ on the intervals $(-7, -4), (0, 3)$ and $(3, 8)$.
- $f''(x) < 0$ on the intervals $(-\infty, -7), (-4, 0)$ and $(8, \infty)$.

(a) (10 points) Determine the interval(s) on which $f(x)$ is increasing and on which $f(x)$ is decreasing AND indicate where $f(x)$ has relative maximum and relative minimum points. (If there are none, please say so).

Answer (6a): increasing: ____________________

decreasing: ____________________

rel. max. points(s) at $x =$ ____________________

rel. min. point(s) at $x =$ ____________________

(b) (8 points) Determine the interval(s) on which $f(x)$ is concave up and on which $f(x)$ is concave down AND indicate where $f(x)$ has inflection point(s). (If there are none, please say so).

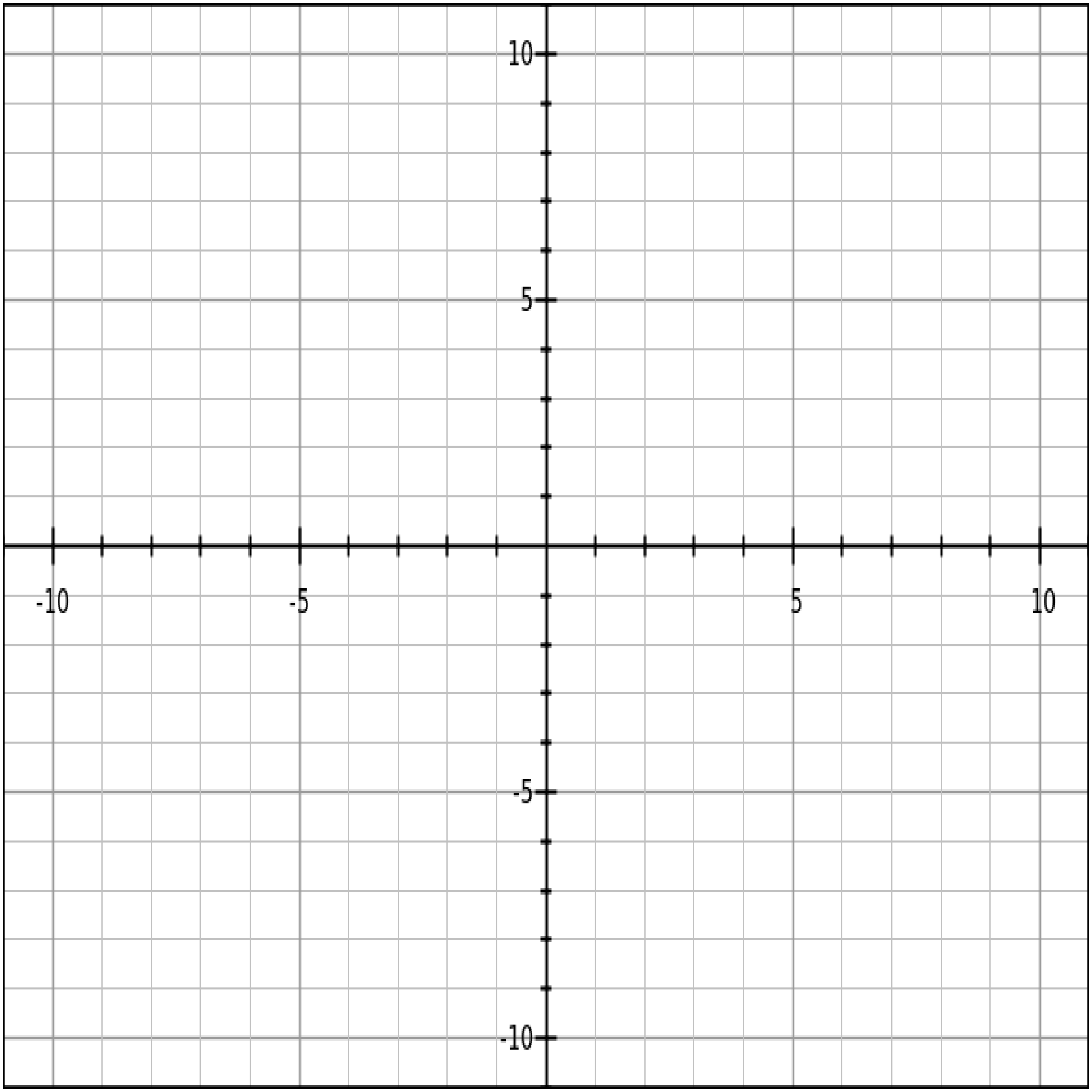
Answer (6b): concave up: ____________________

concave down: ____________________

inflection point(s) at $x =$ ____________________

Problem (6) cont.

(c) (10 points) Sketch a graph of $f(x)$.



(7). (19 points) Find the indefinite or definite integral:

(a) (8 points)

$$\int 5x\sqrt{x^2+9}\,dx$$

Answer (7a): ______________________

(b) (11 points)

$$\int_3^4 \frac{2x}{x^2-8}\,dx$$

Answer (7b): ______________________

(8). (17 points) Suppose a manufacturer's marginal-cost function is

$$\frac{dc}{dq} = 337 + 32q - 6q^2$$

(a) (13 points) Find the total-cost function if the fixed costs are 150.

Answer (8a): Total-cost function: ____________________

(b) (4 points) Determine the change in the manufacturer's total cost if production is increased from 3 to 10 units.

Answer (8b): Change in cost: ____________________

(9). (8 points) Set-up, but DO NOT EVALUATE, an integral to find the area of the region bounded by the given curves. Be sure to find any needed points of intersection.

$$y = x^2 + 3x - 15 \text{ and } y = 5 - 3x - x^2$$

Answer (9): Area: ________________________

(10). (20 points) The demand equation for a product is

$$p = 0.01q^2 - 1.4q + 46$$

and the supply equation is

$$p = 0.01q^2 + 4$$

(a) (5 points) Find the equilibrium point (q_0, p_0).

Answer (10a): $q_0 =$ ____________________

$p_0 =$ ____________________

(b) (15 points) Determine the producers' surplus under market equilibrium.

Answer (10b): Producers' surplus: ____________________