Math 1152 Midterm 1 Important Concepts

The midterm questions will be heavily based upon the following resources:

- The Worksheets available under the "Modules" section in Canvas.
- The sample midterms available on Canvas. Note that the sample midterms are indicative of the style, level of difficulty, and length of the actual midterm and that the format of this semester's exams may be significantly different from exams given previously.
- The previous semesters' midterms available on Canvas.

  Note that by posting these and solutions, it should be clear what the departmental expectations are. However, to encourage you learn concepts and not memorize content, the questions on this semester's exam will require that you use similar logic to the previous semester's midterms. **You should not expect identical questions**!

  Thus, the least important aspect of the old midterm questions is the mechanical technique used to solve a particular question. You should understand the concepts in the solution to a problem so you can employ similar logic to solve a slightly different one!
- Your quizzes. Solutions to these are available on Canvas.

  Note that the solutions to the quizzes contain valuable commentary about how to think about the questions as well as warnings about common errors students make!
- The aspects of Project 1 discussed on your Canvas announcements.

  Algebra and calculus 1 material is essential to working problems correctly in math 1152. The MML Reviews as well as the warnings and commentary on the quiz solutions are designed to sharpen and practice your prerequisite skills, and the exam will give you an opportunity to demonstrate your mastery of these skills!

Recommended Study Plan:

1. Read the Study Guide. If any topic seems challenging, review it in your notes and work a few problems on the worksheets involving it.
2. Work through your old quizzes. DO NOT JUST READ THE SOLUTIONS!
3. Once you feel like you have a good grasp on this material, take one of the sample exams or previous semesters' midterms as if it were your actual exam; schedule 55 uninterrupted minutes and write solutions to the exams without your book or notes. Once you are finished, check your work against the solutions.

   Keep track of any questions you miss. If you miss a particular question:

   - Work through it with the solutions. Once you feel like you understand it, work the problem again without the solutions.
   - Go back to the worksheets and drill that type of question.
   - The next day, try to work the problem again without the solutions.

   Repeat this for the other midterms posted on Canvas.

Any type of question asked on the sample exam, worksheets, and your past quizzes is fair game for the exam. Additionally, the following is a list of important concepts that may arise on the first midterm, broken down by section.

Section 6.2: Area Between Curves

- For a given region, know how to express its area with respect to both x and y.
- For a given region, be able to identify if there is a preferred variable of integration.
- Be able to compute the area of a given region.

Section 6.3: Volume by Slicing

- Know how to set up and compute the volume of solids whose base is a given region in the xy-plane and whose cross-sections, perpendicular to an axis, are given.
- You are expected to know the formulas for areas of squares, rectangles, and semicircles. Any other area formulas you need will be provided if necessary.
- Be able to explain why the disk/washer method is a specific example of volume by slicing.

Section 6.3/6.4: Washer and Shell Method

- Be able to set up an integral or sum of integrals that represents the volume of a solid of revolution using either the shell method or washer method.
- Be able to choose the appropriate method if you are given the variable of integration (i.e. be able to set up an integral or sum of integrals that represents the volume of a solid of revolution as an integral or sum of integrals with respect to x or y).
- Be able to determine which method is more effective to compute a given volume.
- Be able to compute the volumes when regions are revolved about any line parallel to the x or y axis (e.g. $x = 3$ or $y = -2$).
- Be able to approximate the volume of a given region using disks, washers, or shells.

Section 6.5: Arclength

- Know how to set up an integral that represents the length of a given segment of a curve and evaluate it if asked. Note that many integrals that arise here can only be evaluated if you're meticulous with your algebra and differentiation!
- Know how to approximate the length of a segment of a curve.

Section 6.6: Surface Area

- Know how to set up an integral that represents the area of the resulting surface when a given segment of a curve is rotated about a given axis.
- Be able to evaluate a surface area integral if asked. Note that many integrals that arise here can only be evaluated if you're meticulous with your algebra and differentiation!

Section 6.7: Physical Applications

- Know how to find the mass of a segment of a thin wire and how to calculate the work done by a variable force.
- Know Hooke's Law for a spring and be comfortable solving spring problems.
- Know how to find the work required to pump liquid out of a tank.

Chapter 6: A Broader Perspective

Chapter 6 presents the method necessary to solve many problems in the physical sciences: *slice, approximate, integrate.*

Each section, from calculating areas, to volumes of revolution, to lengths of curves, to physical applications, is an example of how this procedure allows one to solve a wide variety of different problems. Though these sections may all ask different types of questions, the underlying methodology used to answer each problem is similar. You will be expected to employ this methodology to solve an unfamiliar problem. See the sample exams and project 1 for examples.

Section 7.1: Basic Integration Techniques

- Know the connection between antidifferentiation and differentiation, i.e. know that if $\int f(x)\,dx = F(x)$ then $f(x) = F'(x)$.
- Be comfortable with the basic techniques of integration in this section, including u-substitution, splitting fractions, etc.
- Know when a *linear* change of variable is helpful when finding an antiderivative:
  - ex: $\int (x-7)\sqrt{2x+1}\,dx$ can be evaluated first letting $u = 2x+1$.
  - ex$_2$: $\displaystyle\int \frac{x^2}{3x+2}\,dx$ can be evaluated first letting $u = 3x+2$.
- Know when splitting fractions is helpful when finding an antiderivative:
  - ex: $\displaystyle\int \frac{x+2}{x^2+4}\,dx$ can be evaluated by writing as $\displaystyle\int \frac{x}{x^2+4}\,dx + \int \frac{2}{x^2+4}\,dx$.
    The first integral can be evaluated using a u-substitution, whereas the second can be evaluated using the formula:
    $$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$$
    This formula will arise again in section 7.5 and Chapter 10; you should know it!
  - ex$_2$: $\displaystyle\int \frac{3x^2+e^{x^3}}{e^{x^3}}\,dx$ can be evaluated by writing as $\displaystyle\int \frac{3x^2}{e^{x^3}}\,dx + \int \frac{e^{x^3}}{e^{x^3}}\,dx$.
    The first integral can be evaluated by writing $\dfrac{3x^2}{e^{x^3}} = 3x^2e^{-x^3}$ then using a u-substitution, whereas the second integral can be evaluated easily.

You should understand why splitting these fractions up is a good idea! If you do not, ask your instructor!

- Be able to do all of the questions from Worksheet 1.
- Be able to evaluate integrals involving common functions whose arguments are linear functions of x; e.g. you should be able to compute the antiderivatives of all of the following functions, and ones line them, instantly without having to do a substitution:

$$\int e^{\frac{x}{4}}\,dx, \qquad \int \cos(5x)\,dx, \qquad \int \sec(3x)\tan(3x)\,dx, \qquad \int \frac{1}{2x+1}\,dx, \qquad \text{etc.}$$

Section 7.2: Integration by Parts

- Know when to use integration by parts to evaluate both definite and indefinite integrals.
- Be able to use integration by parts more than once to evaluate an antiderivative.

Study the sample midterms available on Canvas, your quizzes, as well as the worksheets, to get a good idea of what type of computations you will be expected to do!