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General Information:

This is a midterm from a previous semester. This means:

- This midterm contains problems that are of similar, but not identical, difficulty to those that will be asked on the actual midterm.
- The format of this exam will be similar, but not identical to this midterm.
- This midterm is of similar length to the actual exam.
- Note that there are concepts covered this semester that do *NOT* appear on this midterm. This does not mean that these concepts will not appear on the actual exam! Remember, this midterm is only a sample of what *could* be asked, not what *will* be asked!

How to take this exam:

You should treat this midterm should be as the actual exam. This means:

- "Practice like you play." Schedule 55 uninterrupted minutes to take the sample exam and write answers as you would on the real exam; include appropriate justification, calculation, and notation!
- Do not refer to your books, notes, worksheets, or any other resources.
- You should not need (and thus, should not use) a calculator or other technology to help answer any questions that follow.

  However, in your future professions, you will need to use mathematics to solve many different types of problems. As such, part of the goal of this course is:

  - to develop your ability to understand the broader mathematical concepts (not to encourage you just to memorize formulas and procedures!)
  - to apply mathematical tools in unfamiliar situations (Indeed, tools are only useful if you know when to use them!)

  There have been questions in your online homework and projects with this intent, and there will be a problem on your midterm that will require you to apply the material in an unfamiliar setting.

How to use the solutions:

DO NOT JUST READ THE SOLUTIONS!!!

The least important aspect of the solutions is learning the steps necessary to solve a specific problem. You should be looking for the concepts required to provide solutions. Content may not be recycled, but concepts will be!

- Work each of the problems on this exam *before* you look at the solutions!
- *After* you have worked the exam, check your work against the solutions. If you are miss a type of question on this midterm, practice other types of problems like it on the worksheets!
- If there is a step in the solutions that you cannot understand, please talk to your TA or lecturer!

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Math 1152 Name: Solutions

Midterm 1 OSU Username (name.nn): ____________

Spring 2016 Lecturer: ____________

Recitation Instructor: ____________

Form A Recitation Time: ____________

Instructions

- You have **55 minutes** to complete this exam. It consists of 6 problems on 10 pages including this cover sheet. Page 9 has some potentially useful formulas and both Pages 9 and 10 may be used extra workspace.
- If you wish to have any work on the extra workspace pages considered for credit, indicate in the problem that there is additional work on the extra workspace pages and **clearly label** to which problem the work belongs on the extra pages.
- The value for each question is both listed below and indicated in each problem.
- Please **write clearly** and make sure to **justify your answers** and **show all work**! Correct answers with no supporting work may receive no credit.
- You may not use any books or notes during this exam
- Calculators are NOT permitted. In addition, neither PDAs, laptops, nor cell phones are permitted.
- Make sure to read each question carefully.
- Please **CIRCLE** your final answers in each problem.
- A random sample of graded exams will be copied prior to being returned.

Problem	Point Value	Score
1	20	
2	30	
3	15	
4	20	
5	15	
Total	100	

1. **Multiple Choice** [20 pts]

Circle the response that best answers each question. Each question is worth 4 points. There is no penalty for guessing and no partial credit.

I. The magnitude of the force experienced by a particle moving along the x-axis is given by $F(x) = e^{2x}$.
Find the total work done by this force to move a particle from $x = 0$ to $x = 2$.

A. e^4 B. $e^4 - 1$ C. $2e^4$ D. $2e^4 - 2$ $\boxed{\text{E. None of the above}}$

$$W = \int_0^2 F(x)\,dx = \int_0^2 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_0^2$$
$$= \frac{1}{2}\left[e^4 - e^0\right]$$
$$= \frac{1}{2}e^4 - \frac{1}{2}$$

II. Find a function $f(x)$ that satisfies $\int f(x)\,dx = 4x^3 \cos x + C$.

A. $f(x) = x^4 \sin x$

B. $f(x) = -12x^2 \sin x$

$\boxed{\text{C. } f(x) = 12x^2 \cos x - 4x^3 \sin x}$

D. $f(x) = 12x^2 \cos x - 4x^3 \sin x + C$

E. No such function exists

F. None of the above

If $\int f(x)\,dx = 4x^3 \cos x + C$ then
$$f(x) = \frac{d}{dx}\left[4x^3 \cos x + C\right]$$
$$= 12x^2 \cos x - 4x^3 \sin x.$$

III. A spring requires a force of 60 N to stretch it 2 meters from its equilibrium position. Which of the following describes the magnitude of the additional force, F, required to stretch it an additional 2 meters?

A. $F < 60$ N $\boxed{\text{B. } F = 60 \text{ N}}$ C. $F > 60$ N

D. Not enough information is provided to determine this.

The force required to stretch it 2 additional meters is $\Delta F = k\Delta x$, which is linear in Δx

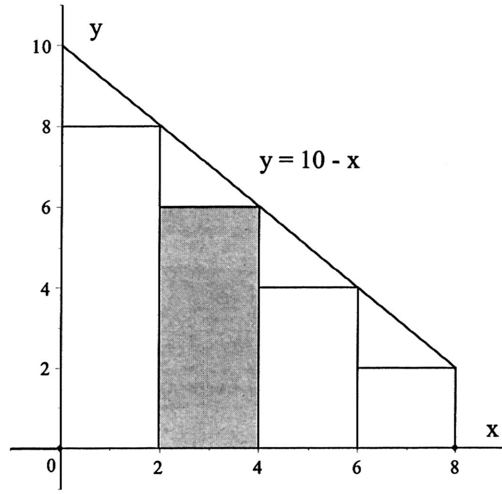
IV. A thin wire is represented by a line segment on the interval $x = 0$ to $x = 10$ and its density is given by $\rho(x) = 3x$. If the center of mass[1] of the wire is at $x = a$, then:

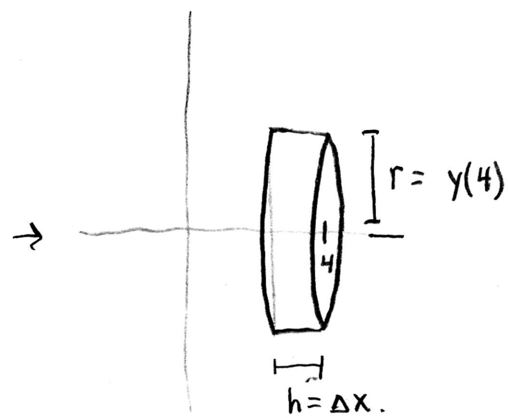
A. $a < 5$ B. $a = 5$ $\boxed{\text{C. } a > 5}$

D. Not enough information is provided to determine this.

The density is increasing in x, so the portion of rod from x=0 to x=5 is less massive than the portion to the right of x=5. The CM must be right of x=5.

V. Let R be the region bounded by $y = 10 - x$, $y = 0$, $x = 0$, and $x = 8$.
When R is revolved about the x-axis, a solid of revolution is formed. This solid can be approximated by slicing the region into 4 rectangles of equal width that coincide with the function $y = 10 - x$ at their righthand endpoints, then revolving these rectangles about the x-axis.





The shape obtained by revolving **the shaded rectangle** about the x-axis has volume:

$\boxed{\text{A. } 72\pi}$ B. 36π C. $\int_0^8 \pi(10-x)^2\,dx$ D. $\int_0^8 2\pi x(10-x)\,dx$

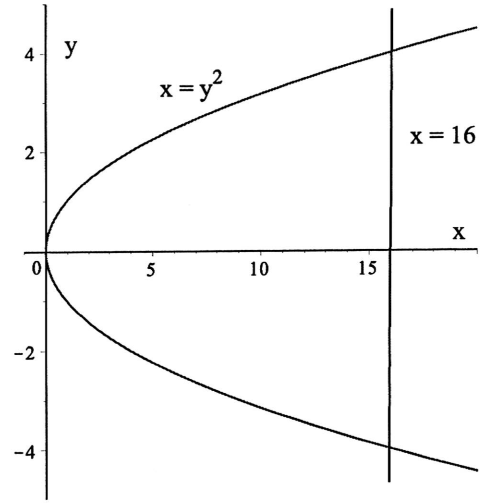
E. None of the above

The shape will be a disk of radius $y(4) = 10 - 4 = 6$ and height $\Delta x = 2$.

The volume is thus $V = \pi r^2 h = \pi(6)^2 \cdot 2 = 72\pi$

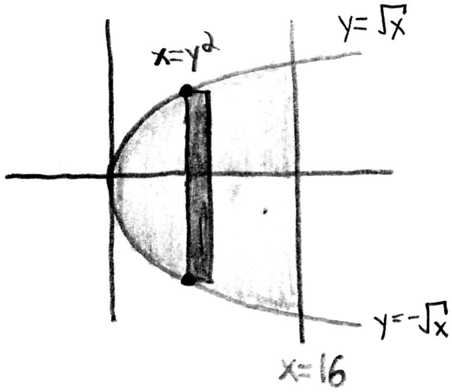
[1] "The center of mass is at $x = a$" means that the mass of the portion of the wire to the left of $x = a$ equals the mass of the wire to the right of $x = a$.

2. [30 pts] The region R is bounded by the curves $x = y^2$ and $x = 16$, shown below:



I. Set up, but do not evaluate, an integral or a sum of integrals **with respect to x** that would give the area of R.

"Integrate wrt x" → <u>use vertical rectangles</u>



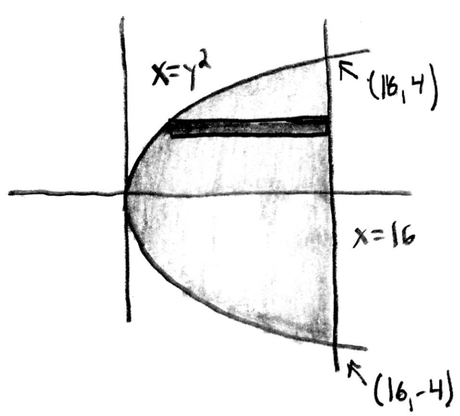
$$A = \int_0^{16} (\text{top} - \text{bottom})\, dx$$

"dx" means <u>everything</u> must be expressed in terms of x!

$$\boxed{A = \int_0^{16} \left[\sqrt{x} - (-\sqrt{x})\right] dx}$$

II. Set up, but do not evaluate, an integral or a sum of integrals **with respect to y** that would give the area of R.

"Integrate with respect to y" → <u>Use horizontal rectangles</u>



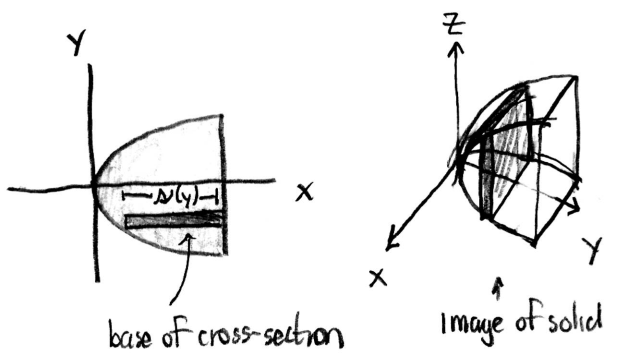
$$A = \int_{-4}^{4} (\text{right} - \text{left})\, dy$$

"dy" means <u>everything</u> must be expressed in terms of y!

$$= \boxed{\int_{-4}^{4} (16 - y^2)\, dy}$$

(Problem 2 continued)

III. The base of a solid is the region R on the previous page. Cross sections through the solid perpendicular to the y−axis are squares. Set up, but do not evaluate, an integral or sum of integrals that gives the volume of the solid.



$V = \int_{-4}^{4} A(y)\, dy$

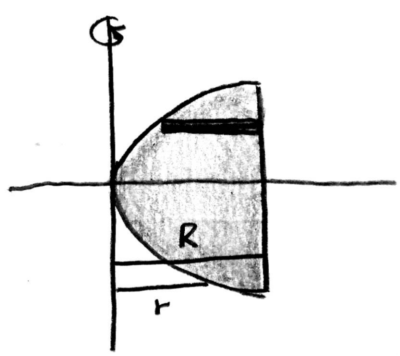
Since the cross-sections are squares,

$A(y) = [s(y)]^2$

$= [16-y^2]^2.$

So: $\boxed{V = \int_{-4}^{4} [16-y^2]^2\, dy}$

IV. A solid of revolution is formed by revolving the region R on the previous page about the y−axis. Set up, but do not evaluate, an integral or sum of integrals **with respect to y** that gives the volume of this solid.



- Integrate wrt y → Use horizontal rectangles
- Horizontal rect are $\perp$ the axis of rotation ⇒ Washer Method

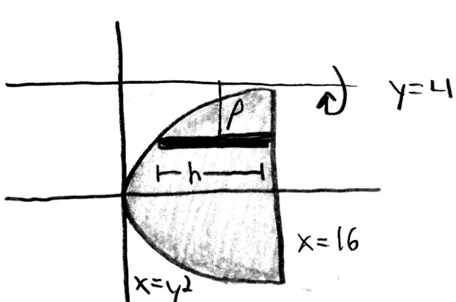
$V = \pi \int_{-4}^{4} [R^2 - r^2]\, dy$

R: dist from axis to outer curve → $R = 16 - 0 = 16$

r: dist from axis to inner curve → $r = y^2 - 0 = y^2$.

→ $\boxed{V = \int_{-4}^{4} \pi [(16)^2 - (y^2)^2]\, dy}$

V. A solid of revolution is formed by revolving the region R on the previous page about the line $y = 4$. Set up, but do not evaluate, an integral or sum of integrals that gives the volume of this solid.



- We will still integrate wrt y and use horizontal rect.
- Horizontal rect are ∥ axis of rotation → Use Shell Method

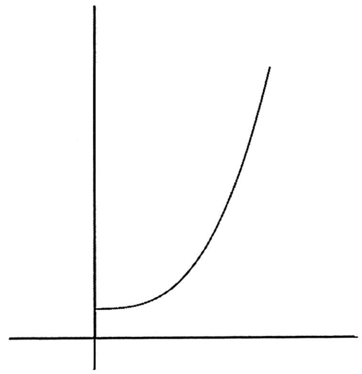
$V = 2\pi \int_{-4}^{4} ph\, dy.$

p: distance from axis to rectangle → $p = 4 - y$

h: height of rectangle → $h = 16 - y^2$

→ $\boxed{V = \int_{-4}^{4} 2\pi (4-y)(16-y^2)\, dy}$

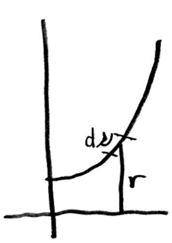
3. [15 pts] The segment of the curve $y = 1 + 8x^3$ from $x = 0$ to $x = 1$ is shown below:



I. [3 pts] Set up, but do not evaluate, an integral that gives the length of this segment.

$$s = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \boxed{\int_0^1 \sqrt{1 + (24x^2)^2}\, dx}$$

II. [6 pts] Set up, but do not evaluate, and integral that expresses the area of the surface generated when this segment is revolved **about the x-axis.**



$SA = \int_a^b 2\pi r\, ds$ where r is the distance from the axis to the segment

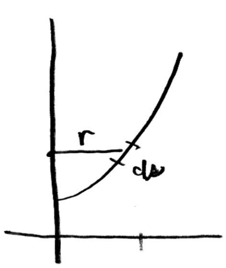
$ds = \sqrt{1+(y')^2}\, dx$ or $\sqrt{1+(x')^2}\, dy$ (your choice!)

Here, it is nice to integrate wrt x, so use $ds = \sqrt{1+(y')^2}\, dx$,

and write r in terms of x: $r = (1+8x^3) - 0 = 1+8x^3$.

So: $\boxed{SA = \int_0^1 2\pi\,(1+8x^3)\sqrt{1+(24x^2)^2}\, dx}$

III. [6 pts] Set up, but do not evaluate, and integral that expresses the area of the surface generated when this segment is revolved **about the y-axis.**



We still would prefer to integrate wrt x, so let $ds = \sqrt{1+(y')^2}\, dx$

and write r in terms of x: $r = x$.

So: $\boxed{SA = \int_0^1 2\pi x \sqrt{1+(24x^2)^2}\, dx}$

4. [20 pts] Evaluate the following antiderivatives.

I. $\displaystyle\int_0^{\pi} 4x\cos(2x)\,dx$

$u = 4x \qquad dv = \cos 2x\,dx$

$du = 4\,dx \qquad v = \frac{1}{2}\sin 2x$

$$\int_0^{\pi} 4x\cos 2x\,dx = \Big[\overset{u}{4x}\cdot\overset{v}{\tfrac{1}{2}\sin 2x}\Big|_0^{\pi} \;\overset{-}{-}\; \overset{\int v}{\int_0^{\pi}\tfrac{1}{2}\sin 2x}\cdot\overset{du}{4dx}$$

$$= 2\pi\,\overset{0}{\cancel{\sin 2\pi}} - 0 - \cos 2x\Big|_0^{\pi}$$

$$= -\cos 2\pi + \cos 0$$

$$= \boxed{0}$$

II. $\displaystyle\int \frac{9e^{3x}}{18+2e^{6x}}\,dx$

Hint: First, make the substitution $u = e^{3x}$.

$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\dfrac{du}{3e^{3x}} = dx$

So $$\int \frac{9e^{3x}}{18+2e^{6x}}\,dx = \int \frac{9\cancel{e^{3x}}}{18+2e^{6x}}\,\frac{du}{3\cancel{e^{3x}}}$$ ← Need e^{6x} in terms of $u = e^{3x}$

$$= \int \frac{3}{2[9+(e^{3x})^2]}\,du$$

$$= \frac{3}{2}\int \frac{1}{9+u^2}\,du$$

$$= \frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{3}}\arctan\frac{u}{3} + C$$

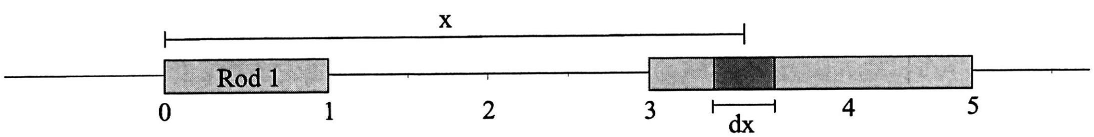
$$= \boxed{\frac{1}{2}\arctan\left(\frac{e^{3x}}{3}\right) + C}$$

5. [15 pts] **Read the following problem closely!**

The force that a rod of mass M of uniform density and length L exerts on a particle aligned with it of mass m is given by:

$$F = \frac{GMm}{x(x-L)}.$$

where x is the distance between the particle and the (farther) edge of the rod and G is a constant. Now suppose there are two rods 2 units apart that are aligned with each other, as shown below:



Suppose Rod 1 has mass $M = 2$ and length $L = 1$ and that Rod 2 has constant linear density ρ and length 2. The total force that Rod 1 exerts on Rod 2 cannot be found using the given force equation because the magnitude of the force exerted by Rod 1 on different segments of Rod 2 is different!

However, the force exerted by Rod 1 on a small segment of Rod 2 of length dx (which you may call Δx if you prefer), centered at a value x on Rod 2, can be approximated by treating the small segment as a particle.

I. Express the mass of the small segment in terms of ρ and dx (or Δx).

$$\Delta m = \rho \Delta x$$

II. Express the approximate force that Rod 1 exerts on the small segment.

$$\Delta F = \frac{GM\,\Delta m}{x(x-L)} = \frac{G(2)\cdot \rho \Delta x}{x(x-1)} = 2\rho G \frac{\Delta x}{x(x-1)}.$$

III. Write down an integral that represents the force that the Rod 1 exerts on Rod 2.

$$\boxed{F = \int_3^5 2\rho G \frac{dx}{x(x-1)}}$$

For the lower limit, use the x position of the closest segment of rod 2
upper limit, " " " " " " farthest " " " "