Math 1150
Autumn 2012

Midterm 3
Form A

Name: _____
OSU user name (name.nn): _____
Recitation Instructor: _____
Recitation Time: _____

The point value of each problem is indicated. To obtain full credit you must have the correct answers along with **the supporting work**. Answers without supporting work will receive no credit, except for multiple choice problems. **CIRCLE YOUR ANSWERS**.

1. (16 points) Multiple Choice: **Circle your answer.**

   (a) The expression $\cos(x)\cos(60) + \sin(x)\sin(60)$ equals:

   (i) $\cos(x + 60)$ (ii) $\cos(x - 60)$ (iii) $\sin(x + 60)$ (iv) Not listed

   (b) The vectors $\mathbf{u} = < 8, 6 >$ and $\mathbf{v} = < -15, 20 >$ are perpendicular.

   (i) Yes (ii) No

   (c) Simplify $\tan(x + \frac{5\pi}{4})$:

   (i) $-\tan(x)$ (ii) $\frac{1 + \tan(x)}{1 - \tan(x)}$ (iii) $\frac{1 - \tan(x)}{1 + \tan(x)}$ (iv) Not listed

   (d) For any two vectors $\mathbf{u}$ and $\mathbf{v}$, the component of $\mathbf{u}$ along $\mathbf{v}$ is equal to the component of $\mathbf{v}$ along $\mathbf{u}$.

   (i) True (ii) False

2. (20 points)

(a) The solutions of the trigonometric equation $\tan(3\theta) + 1 = 0$ are:

(i) $\frac{3\pi}{4} + k\pi$ (ii) $\frac{\pi}{4} + k\pi$ (iii) $\frac{\pi}{4} + \frac{k\pi}{3}$ (iv) Not listed

where $k$ is an integer.

(b) Write $\sin(2\sin^{-1}(3x))$ in terms of $x$ only.

(i) $6x\sqrt{1-9x^2}$ (ii) $6x$ (iii) $\sqrt{1-9x^2}$ (iv) Not listed

(c) The angle between the vectors $\mathbf{u} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} - 4\mathbf{j}$ is approximately:

(i) $95^\circ$ (ii) $27^\circ$ (iii) $153^\circ$ (iv) Not listed

(d) Two forces $F_1 = < -3, 1 >$ and $F_2 = < 2, 5 >$ act on an object. The magnitude of the resultant force is:

(i) $\sqrt{37}$ (ii) $\sqrt{35}$ (iii) 37 (iv) Not listed

3. (24 points)

a) (8 points) Find all solutions of the trigonometric equation $\tan(2\theta)\sin(\theta)+\sin(\theta) = 0$.

b) (8 points) An ellipse has an equation $3x^2 + 5y^2 = 15$.

i) Find its vertices.

**vertices:** (____,____)

(____,____)

ii) Find its foci.

**foci:** (____,____)

(____,____)

c) (8 points) Simplify the trigonometric expression $\left(1 - \dfrac{1}{\csc(x)}\right)^2 + \cos^2(x)$, and express your answer in terms of $\sin(x)$.

4. (20 points)

a) (10 points) Find the complete solution of the linear system by first eliminating the variables x and y from the last equation. Show your work.

$$\begin{aligned} x - y + 3z &= 4 \\ y - z &= -2 \\ -x + y + z &= 0 \end{aligned}$$

b) (10 points) Evaluate $\sin(\theta - \phi)$ if $\cos\theta = \frac{1}{3}$, $\theta$ is in quadrant IV, $\sin\phi = \frac{2}{3}$, and $\cos\phi = \frac{\sqrt{5}}{3}$.

5. (20 points)

(a) A force $\mathbf{F}$ has magnitude $|\mathbf{F}| = 50$ and direction $\theta = 120°$. Find its horizontal and vertical components.

(i) $\mathbf{F} = < -25, 25\sqrt{3} >$

(ii) $\mathbf{F} = < 25, 25\sqrt{3} >$

(iii) $\mathbf{F} = < -25\sqrt{3}, 25 >$

(iv) Not listed

(b) Write the complex number $z = -3 + 5i$ in polar form with angle $\theta$ between 0 and $360°$.

(i) $\sqrt{34}(\cos(121°) + i\sin(121°))$

(ii) $34(\cos(121°) + i\sin(121°))$

(iii) $\sqrt{34}(\cos(59°) + i\sin(59°))$

(vi) Not listed

(c) Compute $z^{-20}$ where $z$ is the complex number $6(\cos 70° + i\sin 70°)$.

(i) $6^{20}(\cos 40° + i\sin 40°)$

(ii) $\dfrac{1}{6^{20}}(\cos 40° + i\sin 40°)$

(iii) $\dfrac{1}{6^{20}}(\cos 320° + i\sin 320°)$

(iv) Not listed

(d) Find $\cos(\dfrac{x}{2})$ if $\sec(x) = \dfrac{5}{2}$; $270° < x < 360°$.

(i) $\dfrac{-\sqrt{7}}{2}$

(ii) $-\sqrt{\dfrac{7}{10}}$

(iii) $\sqrt{\dfrac{7}{10}}$

(iv) Not listed

# Formulas

- $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
- $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
- $\tan(x+y) = \dfrac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$
- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$
- $\sin(\dfrac{x}{2}) = \pm\sqrt{\dfrac{1-\cos(x)}{2}}$
- $\cos(\dfrac{x}{2}) = \pm\sqrt{\dfrac{1+\cos(x)}{2}}$
- $a + bi = r(\cos(\theta) + i\sin(\theta))$ where $r = \sqrt{a^2+b^2}$ and $tan(\theta) = \dfrac{b}{a}$
- Parabola with focus $(0, p)$ and vertex $(0, 0)$:

$$x^2 = 4py$$

- Ellipse with foci $(\pm c, 0)$ and vertices $(\pm a, 0)$:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad (a > b > 0)$$

$$c^2 = a^2 - b^2;$$

$$\text{Eccentricity: } e = \frac{c}{a}$$

- Hyperbola with foci $(\pm c, 0)$ and vertices $(\pm a, 0)$:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad (a > 0, b > 0)$$

$$c^2 = a^2 + b^2;$$

$$\text{Asymptotes: } y = \pm\frac{b}{a}x$$