Math 1151
MIDTERM 3
December 1, 2015
Form A

NAME : ___ ___ ___ ___ ___ ___ ___ ___

OSU Name.# : ___ ___ ___ ___ ___ ___ __

Lecturer:: ___ ___ ___ ___ ___ ___ ___

Recitation Instructor : ___ ___ ___ ___

Recitation Time : ___ ___ ___ ___ ___ __

INSTRUCTIONS

- **SHOW ALL WORK** in problems 1 and 4 .
  Incorrect answers with work shown may receive partial credit,
  but unsubstantiated correct answers may receive NO credit.

  You don ' t have to show work in problems 2 and 3 .

- Give EXACT answers unless asked to do otherwise.

- Calculators are NOT permitted !
  PDA ' s , laptops, and cell phones are prohibited.
  Do not have these devices out !

- The exam duration is 55 minutes.

- The exam consists of 4 problems starting on page 2 and ending on page 8.
  Make sure your exam is not missing any pages before you start.

PROBLEM NUMBER	SCORE
1	(46)
2	(18)
3	(18)
4	(18)
TOTAL	(100)

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1. (46 pts) SHOW YOUR WORK !

(I) (10 pts) Evaluate the limit . You may use L ' Hospital ' s Rule.

$$\lim_{x \to 0} \left(\frac{\cos x - 1 + x^2}{x^2} \right)$$

(II) (12 pts)

The limit of Riemann sums for a function f on the interval [1, 5] is given by

$$\lim_{\Delta \to 0} \sum_{k=1}^{n} \left(2\, x_k^* + \frac{1}{x_k^*} \right) \Delta x_k \qquad \text{on } [1,\ 5].$$

(a) Identify f and express the limit as a definite integral.

(b) Evaluate the limit of Riemann sums.

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1. (46 pts) SHOW YOUR WORK !

(III) (12 pts)
Given the acceleration function of an object moving along a line, find the position function with the given initial velocity and position.

$$a(t) = 2t, \quad v(0) = 2, \quad s(0) = 0$$

(IV) (12 pts)

(a) **Find the linearization, L (x), of the function**

$$f(x) = e^{2x} \quad \text{at } a = 0.$$

(b) **Using the linearization, L (x), from the part (a), approximate e.**

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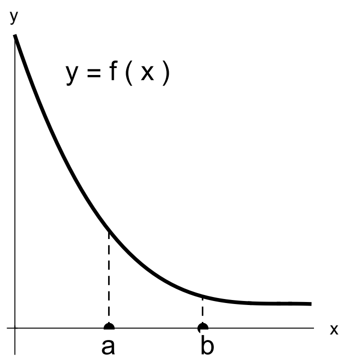
2. (18 pts) **MULTIPLE CHOICE !**

(I)

The figure shows the graph of a function f.

Let $L_a(x)$ be the linear approximation of f at a.



Circle ALL the correct statements below.

(a) $L_a(b) < f(b)$; (b) $L_a(b) > f(b)$; (c) $L_a(a) < f(a)$;

(d) $L_a(a) > f(a)$; (e) No statement (a) - (d) is correct.

(II)

Let g be a continuous function on [3, 5], such that

$g(3) = 2$, $g(5) = 8$, and

$g'(x) > 0$ for all x in (3, 5).

Circle ALL the correct statements below.

(a) $0 < g'(x) \leq 2$, for all x in (3, 5);

(b) $0 < g(x) \leq 8$, for all x in (3, 5);

(c) $g'(c) = 3$, for some number c in (3, 5);

(d) g does not satisfy the conditions of the Mean Value Theorem on [3, 5];

(e) No statement (a) - (d) is correct.

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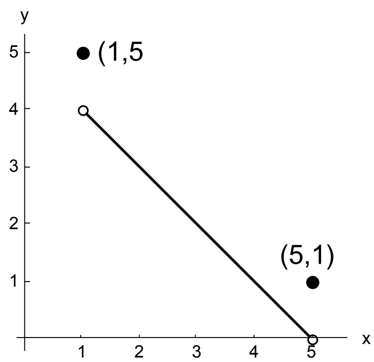
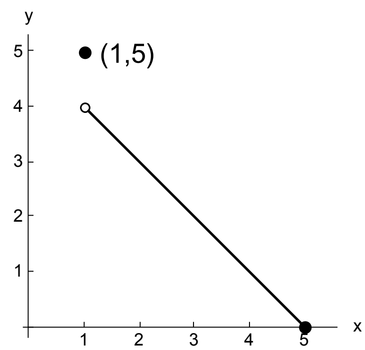
2. (18 pts) MULTIPLE CHOICE !

(III) Given the four functions on interval [1, 5], answer the questions below.

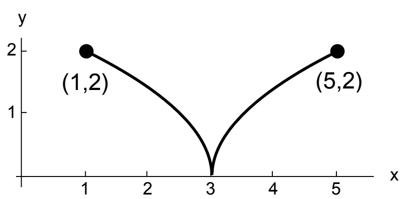
(A)



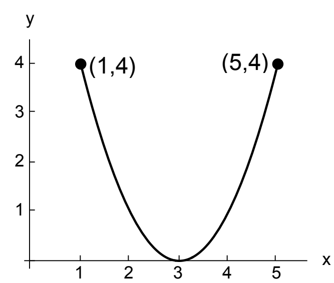
(B)



(C)



(D)



(i)

Circle the function that satisfies (or functions that satisfy) the conditions of the Mean Value Theorem on [1, 5].

A **B** **C** **D**

(ii)

Circle the function (or functions) for which there exists a point c in (1, 5) such that $f'(c) = \dfrac{f(5) - f(1)}{5 - 1}$.

A **B** **C** **D**

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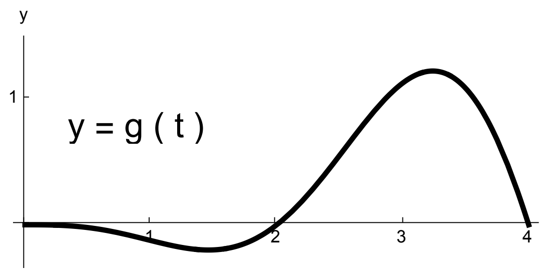
3. (18 pts) MULTIPLE CHOICE ! CIRCLE THE CORRECT ANSWER IN EACH PART.

The graph of g, a continuous function on [0, 4], is shown in the figure.

Let
$A(x) = \int_0^x g(t)\, dt$, for $0 \le x \le 4$.



(I) (3 pts) **Circle the correct statement about A (2).**

(a) $A(2) = 0$; (b) $A(2) > 0$; (c) $A(2) < 0$;

(e) **NONE OF THE PREVIOUS ANSWERS.**

(II) (3 pts) **Circle the correct statement about A (3.8).**

(a) $A(3.8) = 0$; (b) $A(3.8) > 0$; (c) $A(3.8) < 0$;

(e) **NONE OF THE PREVIOUS ANSWERS.**

(III) (3 pts) **Circle the correct statement about A ' (3.8).**

(a) $A'(3.8) = 0$; (b) $A'(3.8) > 0$; (c) $A'(3.8) < 0$;

(e) **NONE OF THE PREVIOUS ANSWERS.**

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3. (18 pts) MULTIPLE CHOICE ! CIRCLE THE CORRECT ANSWER IN EACH PART.

(IV) (3 pts)

Find the solution of the following initial value problem :

$$y'(x) = g(x); \quad y(0) = 2$$

(a) $y(x) = g(x)$; (b) $y(x) = g(x) + 2$;

(c) $y(x) = A(x)$; (d) $y(x) = A(x) + 2$;

(e) $y(x) = g'(x)$; (f) $y(x) = g'(x) + 2$;

(g) **NONE OF THE PREVIOUS ANSWERS.**

(V) (3 pts) **Find the expression for** $\int_0^4 |g(t)|\, dt$.

(a) $A(4)$; (b) $A(2) - A(4)$; (c) $A(4) - A(2)$;

(d) $A(4) - 2A(2)$; (e) **NONE OF THE PREVIOUS ANSWERS.**

(VI) (3 pts) **Find the midpoint Riemann sum for the function g on the interval [0, 4] with n = 2 (the number of subintervals).**

(a) $g(1) + g(3)$; (b) $g(0) + g(2)$; (c) $g(2) + g(4)$;

(d) $2g(1) + 2g(3)$; (e) $2g(0) + 2g(2)$; (f) $2g(2) + 2g(4)$;

(g) **NONE OF THE PREVIOUS ANSWERS.**

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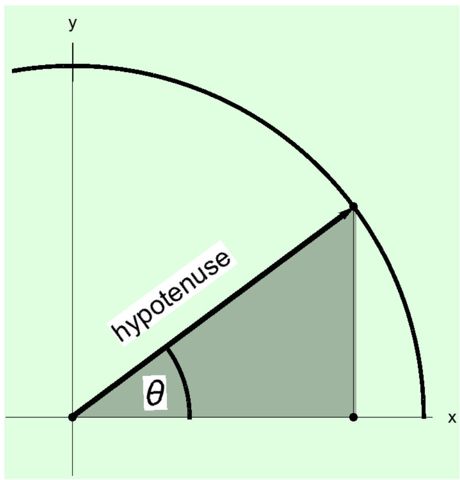
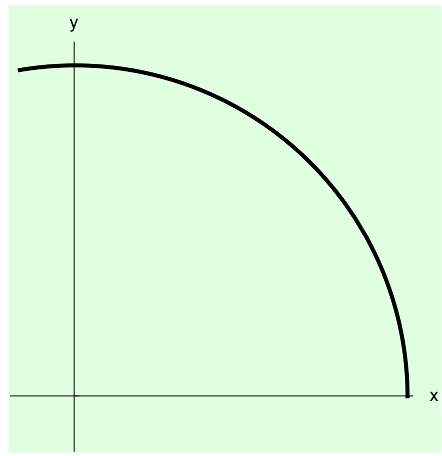
4. (18 pts) **A part of a circle centered at the origin with radius r = 7 cm is given in the figure (A) below.**
A right triangle is formed in the first quadrant (see figure (A)).
One of its sides lies on the x - axis.
Its hypotenuse runs from the origin to a point on the circle.
The hypotenuse makes an angle θ with the x - axis.

(A)



(B)



Make sure to label the picture.

(a) **Draw 2 more examples of such a triangle in the figure (B).**

(b) **Express the area of such a triangle as a function of θ and state its domain.**

A (θ) =

Domain of A =

(c) **Find the value of θ which maximizes the area in part (b). Show your work and justify your answer.**