NAME : ___ ___ ___ ___ ___ ___ ___ ___

OSU Name.# : ___ ___ ___ ___ ___ ___ __

Lecturer:: ___ ___ ___ ___ ___ ___ ___

Recitation Instructor : ___ ___ ___ ___

Recitation Time : ___ ___ ___ ___ ___ __

## INSTRUCTIONS

- **SHOW ALL WORK** in problems 1, 3, 4, and 5 .
  Incorrect answers with work shown may receive partial credit,
  but unsubstantiated correct answers may receive NO credit.

  You don't have to show work in problem 2.

- Give EXACT answers unless asked to do otherwise.
  Include units in your answer where that is appropriate.

- You do not need to simplify numerical answers such as $\frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}}$.

- Calculators are NOT permitted!
  PDA's, laptops, and cell phones are prohibited.
  Do not have these devices out!

- The exam duration is 55 minutes.

- The exam consists of 5 problems starting on page 2 and ending on page 8.
  Make sure your exam is not missing any pages before you start.

| PROBLEM NUMBER | SCORE |
|---|---|
| 1 | (30) |
| 2 | (16) |
| 3 | (20) |
| 4 | (18) |
| 5 | (16) |
| TOTAL | (100) |

**MIDTERM 2**
**Form A, Page 2**

**1. (30 pts)**

**(I) Calculate the derivatives of the following functions.**

**NO NEED TO SIMLIFY !**

(a) (8 pts) $y = \sqrt{x}\,\tan^{-1}(x^5)$

(b) (8 pts) $y = x^{\sin x}$

**MIDTERM 2**
**Form A, Page 3**

**(II) (14 pts) The curve with equation** $x^2 + xy + y^2 = 4$ **is a "rotated ellipse".**

**(a) Use implicit differentiation to find the derivative** $\frac{dy}{dx}$ **.**

**(b) Consider the two points (3, -1) and (2, -2). Show that one of these points lies on the ellipse defined above, and one does not.**

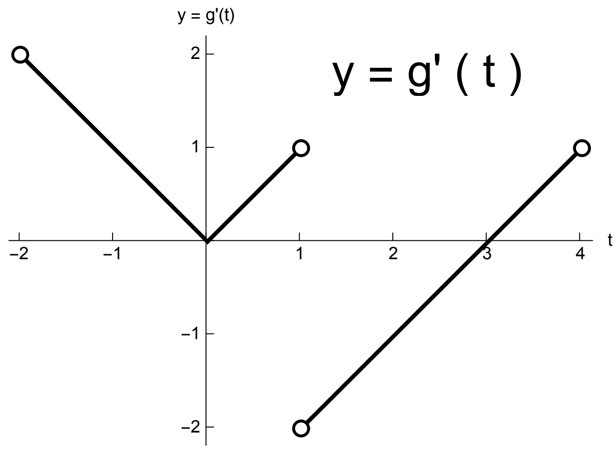
**(c) For the point in part (b) which lies on the ellipse, find the equation of the line tangent to the ellipse at this point.**

**MIDTERM 2**
**Form A, Page 4**

**2.** (16 pts) **EXPLANATION IS NOT REQUIRED !**

**The graph of a function $g'(t)$, the <u>derivative</u> of g is given below.**

**Assume that the function g is continuous on its domain $(-2, 4)$.**




**(I)** **Find all critical points of g.**

**(II)** **On what interval (or intervals) is the function g increasing?**

**MIDTERM 2**
**Form A, Page 5**

**2. (CONTINUED)**

**(III)** **At what point (or points) does the function g have a LOCAL MAXIMUM?**

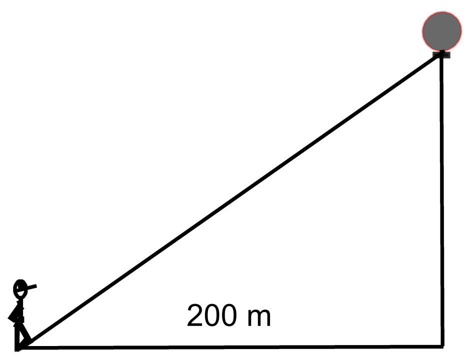
**(IV)** **On what interval (or intervals) is the function g CONCAVE UP?**

**(V)** **Find all inflection points of g.**

**MIDTERM 2**
**Form A, Page 6**

**3. (20 pts)**

**An observer stands 200 meters from the launch site of a hot air balloon.
The balloon rises vertically at a constant rate of 5 m / s.
How fast is the distance between the observer and the balloon changing
20 seconds after the launch?**




**Make sure to label the figure!**

**MIDTERM 2**
**Form A, Page 7**

**4. (18 pts) Show your work.**

**Suppose $s(t) = t^2 + 5$ is the position of an object moving along a line at time $t \geq 0$.**

**(I) Find the instantaneous velocity, $v_{inst}(t)$, at $t = 2$.**

**(II) Alternatively, we can find $v_{inst}(2)$ by following the steps below :**

**STEP 1 : Find the average velocity, $v_{av}(h)$,**

**over the time interval** $\begin{cases} [2+h,\ 2], & \text{if } -1 < h < 0; \\ [2,\ 2+h], & \text{if } 0 < h < 1. \end{cases}$

$v_{av}(h) =$

**STEP 2 : Using the result in step 1, compute the limit. Show your work !**

$\lim_{h\to 0} v_{av}(h) =$

**STEP 3 : Using the result in step 2, find $v_{inst}(2)$.**

**MIDTERM 2**
**Form A, Page 8**

**5. (16 pts) DO NOT USE GRAPHS TO JUSTIFY YOUR ANSWER !**
**NO CREDIT WITHOUT WORK !**

**Let $f(x) = 5 + x \ln x$**

**(a) (8 pts) Find the critical points of f.**

**(b) (8 pts) Find the absolute minimum and the absolute maximum values of f on the interval [1, e]. Justify your answer !**