------------------ DISCLAIMER ------------------

General Information:

This midterm is a *sample* midterm. This means:

- The sample midterm contains problems that are of similar, but not identical, difficulty to those that will be asked on the actual midterm.
- The format of this exam will be similar, but not identical to the actual midterm. Note that this may be a departure from the format used on exams in previous semesters!
- The sample midterm is of similar length to the actual exam.
- Note that there are concepts covered this semester that do *NOT* appear on this midterm. This does not mean that these concepts will not appear on the actual exam! Remember, this midterm is only a sample of what *could* be asked, not what *will* be asked!

How to take the sample exam:

The sample midterm should be treated like the actual exam. This means:

- "Practice like you play." Schedule 55 uninterrupted minutes to take the sample exam and write answers as you would on the real exam; include appropriate justification, calculation, and notation!
- Do not refer to your books, notes, worksheets, or any other resources.
- You should not need (and thus, should not use) a calculator or other technology to help answer any questions that follow.
- The problems on this exam are mostly based on the Worksheets posted on the Math 1152 website and your previous quizzes.

  However, in your future professions, you will need to use mathematics to solve many different types of problems. As such, part of the goal of this course is:

  - to develop your ability to understand the broader mathematical concepts (not to encourage you just to memorize formulas and procedures!)
  - to apply mathematical tools in unfamiliar situations (Indeed, tools are only useful if you know when to use them!)

  There have been questions in your online homework and take-home quizzes with this intent, and there will be a problem on your midterm that will require you to apply the material in an unfamiliar setting. To aid in preparation, there is such a problem on this sample exam.

How to use the solutions:

- Work each of the problems on this exam *before* you look at the solutions!
- *After* you have worked the exam, check your work against the solutions. If you are miss a type of question on this midterm, practice other types of problems like it on the worksheets!
- If there is a step in the solutions that you cannot understand, please talk to your TA or lecturer!

Math 1152

Sample Midterm 1

Spring 2016

Form A

Name: ______________________

OSU Username (name.nn): ______________________

Lecturer: ______________________

Recitation Instructor ______________________

Recitation Time: ______________________

Instructions

- You have **55 minutes** to complete this exam. It consists of 6 problems on 10 pages including this cover sheet. Pages 9 and 10 may be used extra workspace.
- If you wish to have any work on the extra workspace pages considered for credit, indicate in the problem that there is additional work on the extra workspace pages and **clearly label** to which problem the work belongs on the extra pages.
- The value for each question is both listed below and indicated in each problem.
- Please **write clearly** and make sure to **justify your answers** and **show all work!** Correct answers with no supporting work may receive no credit.
- You may not use any books or notes during this exam
- Calculators are NOT permitted. In addition, neither PDAs, laptops, nor cell phones are permitted.
- Make sure to read each question carefully.
- Please **CIRCLE** your final answers in each problem.
- A random sample of graded exams will be xeroxed prior to being returned.

Problem	Point Value	Score
1	20	
2	10	
3	20	
4	15	
5	20	
6	15	
Total	100	

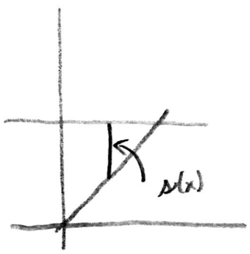
IV. The region in the xy-plane bounded by the lines $y = x$, $x = 0$ and $y = 1$ is the base of two different solids. The first solid has cross-sections perpendicular to the x-axis that are squares while the second solid has cross sections perpendicular to the x-axis that are semicircles. Let V_1 be the volume of the first solid and V_2 be the volume of the second solid. Which of the following best describes the relationship between V_1 and V_2?

A. $V_1 < V_2$ B. $V_1 = V_2$ $\boxed{\text{C. } V_1 > V_2}$

D. Not enough information is provided to determine this.

$V = \int_a^b A(x)\,dx$: For a square: $A_s(x) = [s(x)]^2$,

semicircle $A_c(x) = \frac{1}{2}\pi[r(x)]^2$



Since $r(x) = \frac{1}{2}s(x)$ for each x

$A_c(x) = \frac{1}{2}\pi \cdot \frac{1}{4}[s(x)]^2 = \frac{\pi}{8}[s(x)]^2 < [s(x)]^2 = A_s(x)$.

Thus, $V_1 > V_2$ since the area of the square at each x-value is larger than that of the semicircle!

V. Let W_1 be the amount of work required to stretch a spring 1 m from its equilibrium position and W_2 be the amount of work required to stretch the spring an additional 1 m . Which of the following best describes the relationship between W_1 and W_2?

$\boxed{\text{A. } W_1 < W_2}$ B. $W_1 = W_2$ C. $W_1 > W_2$

D. Not enough information is provided to determine this.

$W = \int_a^b kx\,dx = \frac{1}{2}kx^2 \Big|_a^b = \frac{1}{2}k(b^2 - a^2)$

1. **Multiple Choice** [20 pts]

Circle the response that best answers each question. Each question is worth 4 points. There is no penalty for guessing and no partial credit.

I. A spring has spring constant $k = 20$ N/m. The amount of work required to stretch the spring $3m$ from its equilibrium position is:

A. 60 J

B. 900 J

C. 100 J

D. dependent on whether the spring is stretched or compressed.

$\boxed{\text{E. None of the above}}$

$$W = \int_0^3 F(x)\,dx = \int_0^3 20x\,dx$$

(Hooke's Law: $F(x) = kx$ for springs)

$$= 10x^2\Big|_0^3$$

$$= \underline{90}$$

II. What is the mass of the wire from $x = 0$ to $x = 5$ whose density is given by:

$$\rho(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 9, & 3 < x \le 5 \end{cases}$$

A. $\frac{125}{3}$ units

B. 45 units

$\boxed{\text{C. 27 units}}$

D. None of the above

$$m = \int_0^5 \rho(x)\,dx = \int_0^3 x^2\,dx + \int_3^5 9\,dx$$

$$= \frac{1}{3}x^3\Big|_0^3 + 9x\Big|_3^5$$

$$= \left[\frac{1}{3}(3)^3 - \frac{1}{3}(0)^3\right] + \left[9(5) - 9(3)\right] = \underline{27}$$

III. Which of the following integrals represents the area when the segment of the curve $y = x^2$ from $x = 1$ to $x = 3$ is revolved about the y-axis?

A. $\displaystyle\int_1^3 2\pi y\sqrt{1+4x}\,dy$

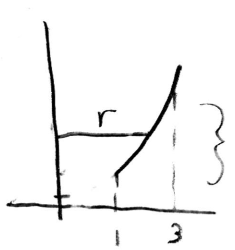
B. $\displaystyle\int_1^9 2\pi y\sqrt{1+y}\,dy$

C. $\displaystyle\int_1^3 2\pi x\sqrt{1+4x}\,dx$

D. $\displaystyle\int_1^9 2\pi y\sqrt{1+\frac{1}{4y}}\,dy$

$\boxed{\text{E. None of the above}}$

$$A = \int_1^3 2\pi x\sqrt{1+4x^2}\,dx \quad \text{or} \quad \int_1^9 2\pi\sqrt{y}\sqrt{1+\frac{1}{4y}}\,dy$$



$SA = \int_{u=a}^{u=b} 2\pi r\,ds$

$r = x.$

2. **Short Answer**

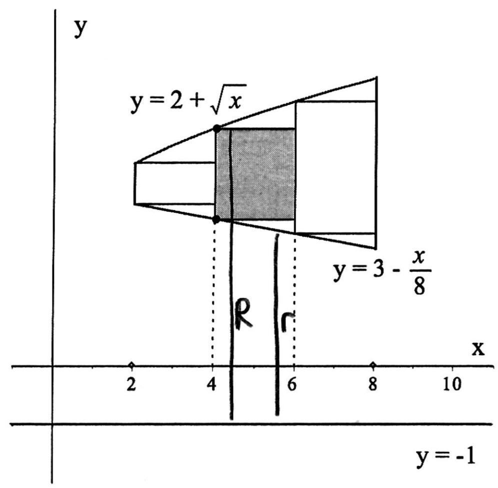
I. [5 pts] Find a function $f(x)$ such that $\int x f(x)\,dx = \cos(x^2) + C$. Justify your response!

$\int x f(x)\,dx = \cos(x^2) + C \leftrightarrow x f(x) = \frac{d}{dx}\left[\cos(x^2) + C\right]$

$x f(x) = -2x \sin(x^2)$

$\boxed{f(x) = -2\sin(x^2)}$

II. [5 pts] Let R be the region bounded by $y = 2 + \sqrt{x}$, $y = 3 - \frac{x}{8}$, $x = 2$, and $x = 8$.
When R is revolved about the line $y = -1$, a solid of revolution is formed. This solid can be approximated by slicing the region into 3 rectangles of equal width that coincide with the functions at their lefthand endpoints, then revolving those rectangles about the line $y = -1$.



R: distance from axis to outer curve

r: distance from axis to inner curve.

Both are vertical here, so take $y_{top} - y_{bottom}$!

Find the volume of the shape obtained by revolving **the shaded rectangle** about the line $y = -1$.

The shape is a washer!

$V = \pi(R^2 - r^2)\Delta x$

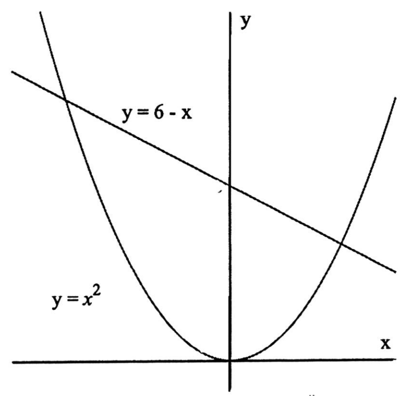
Here $R = (2 + \sqrt{4}) - (-1) = 5$, $\Delta x = 2$

$r = (3 - \frac{4}{8}) - (-1) = \frac{7}{2}$

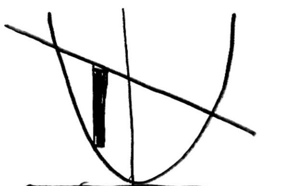
So, $V = \pi\left[(5)^2 - (\frac{7}{2})^2\right] \cdot 2$

$\boxed{V = \frac{51\pi}{2}}$

3. [20 pts] The region R is bounded by the curves $y = x^2$ and $y = 6 - x$, which are shown below:



I. Set up, but do not evaluate, an integral or a sum of integrals **with respect to x** that would give the area of R.



x-limits

$x^2 = 6 - x$

$x^2 + x - 6 = 0$

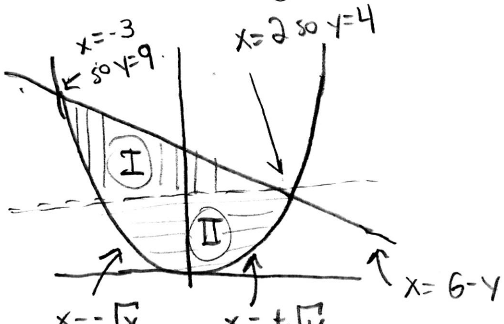
$(x+3)(x-2) = 0$

$x = 2, -3$

$A = \int_{-3}^{2} (\text{top} - \text{bottom})\, dx$

$$\boxed{A = \int_{-3}^{2} [(6-x) - x^2]\, dx}$$

II. Set up, but do not evaluate, an integral or a sum of integrals **with respect to y** that would give the area of R.



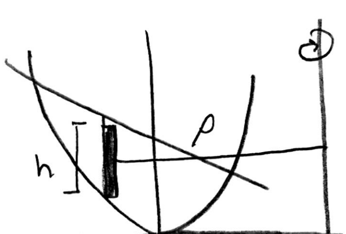
$A_I = \int_4^9 (\text{right} - \text{left})\, dy$

$A_I = \int_4^9 [(6-y) - (-\sqrt{y})]\, dy$

$A_{II} = \int_0^4 [\sqrt{y} - (-\sqrt{y})]\, dy$

$A = A_I + A_{II} \rightarrow \boxed{A = \int_0^4 2\sqrt{y}\, dy + \int_4^9 (6 - y + \sqrt{y})\, dy}$

III. Suppose now that the region R is revolved about the line $x = 6$. Set up, but do not evaluate, an integral or a sum of integrals that would give the volume of the resulting solid.



- We want to integrate with respect to x!

→ Use vertical representative rectangles

- Rect are parallel to x=6

→ Use shell method.

$p = 6 - x$

$h = (6-x) - x^2$

$V = \int_{-3}^{2} 2\pi p h\, dx \rightarrow \boxed{V = \int_{-3}^{2} 2\pi (6-x)[6 - x - x^2]\, dx}$

4. [15 pts] Find the length of the segment of the curve $y = \dfrac{1}{3}(2 + x^2)^{3/2}$ from $x = 0$ to $x = 1$. Simplify your answer completely!

$$l = \int_0^1 \sqrt{1 + (y')^2}\, dx.$$

$$y = \frac{1}{3}(2+x^2)^{3/2}$$

$$y' = \frac{1}{2}(2+x^2)^{1/2} \cdot 2x$$

$$y' = x(2+x^2)^{1/2}$$

$$1+(y')^2 = 1 + x^2(2+x^2)$$

$$= 1+2x^2+x^4$$

$$= (1+x^2)^2$$

So: $$l = \int_0^1 \sqrt{(1+x^2)^2}\, dx$$

$$= \int_0^1 (1+x^2)\, dx$$

$$= \left[x + \frac{1}{3}x^3\right|_0^1$$

$$= \left[1 + \frac{1}{3}(1)^3\right] - 0$$

$$\boxed{l = \frac{4}{3}}$$

5. [20 pts] Evaluate the following antiderivatives.

I. $\displaystyle\int_0^1 \frac{4x+6}{4x^2+1}\,dx$ $= \displaystyle\int_0^1 \frac{4x}{4x^2+1}\,dx + \int_0^1 \frac{6}{4x^2+1}\,dx$

$u = 4x^2+1$

$du = 8x\,dx$

$\frac{du}{8x} = dx$

$\displaystyle\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$

$= \displaystyle\int_{u=1}^{u=5} \frac{4\cancel{x}}{u}\frac{du}{8\cancel{x}} + \int_0^1 \frac{1}{4}\frac{6}{x^2+\frac{1}{4}}\,dx$

$= \frac{1}{2}\ln|u|\Big|_{u=1}^{u=5} + \frac{6}{4}\cdot 2\arctan 2x\Big|_0^1$

$= \frac{1}{2}\ln 5 - \cancel{\frac{1}{2}\ln 1} + 3\arctan 2 - \cancel{3\arctan 0}$

$= \boxed{\frac{1}{2}\ln 5 - 3\arctan 2}$

II. $\displaystyle\int e^{\sqrt{x}}\,dx$

Hint: First, make the substitution $w = \sqrt{x}$.

$w = x^{1/2}$

$dw = \frac{1}{2}x^{-1/2}\,dx$

$dw = \frac{1}{2x^{1/2}}\,dx$

$2x^{1/2}\,dw = dx$

This is w!

$2w\,dw = dx$

So $\displaystyle\int e^{\sqrt{x}}\,dx = \int e^{w}\cdot 2w\,dw$

Now, use integration by parts:

$u = 2w \qquad dv = e^w$

$du = 2\,dw \qquad v = e^w$

So $\displaystyle\int e^{\sqrt{x}}\,dx = 2we^w - \int 2e^w\,dw$

$= 2we^w - 2e^w + C$

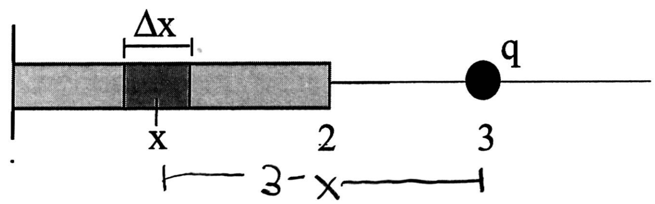
$= \boxed{2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C}$

6. [15 pts] The magnitude of the electric force between two particles with charge q_1 and q_2 is given by Coulomb's Law:

$$F = \frac{kq_1q_2}{r^2}$$

where k is a constant and r is the distance between the two particles.

Suppose now that there is a thin[1] rod that extends from $x = 0$ to $x = 2$ with total charge Q and that this charge is distributed over the rod uniformly. Now, a particle with charge q units is placed at $x = 3$. Follow the steps below to compute the total force that the rod exerts on the particle.



Coulomb's law cannot be applied directly because the magnitude of the force exerted by different segments of the rod on the particle are different! Thus, the rod cannot be treated as a particle!

However, the force exerted by a small segment of the rod of length Δx, centered around a value x between $x = 0$ and $x = 2$ can be approximated by treating the small segment as a particle. By treating the small segment above as if it were a particle:

I. Write an expression for the total charge ΔQ contained in the segment of length Δx (You may call the charge dQ and the segment length dx if you prefer).

$$\Delta Q = \frac{\Delta x}{\text{length of rod}} \cdot Q = \boxed{\frac{Q}{2}\Delta x}$$

II. Write an expression for the approximate force ΔF that the small segment exerts on the particle of charge q at $x = 3$ (You may call the force dF if you prefer).

$$\Delta F = \frac{k(\Delta Q)q}{(3-x)^2} = \boxed{\frac{kQq}{2(3-x)^2}\Delta x}$$

III. Write down an integral that gives the total force, F, that the rod exerts on the particle.

$$\boxed{F = \int_0^2 \frac{kQq}{2(3-x)^2}\,dx}$$

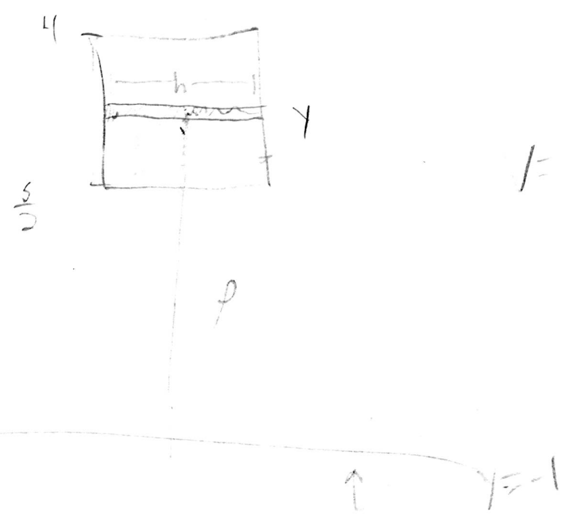
The segments range from x = 0 to x = 2!

[1]'Thin' means that we can neglect any forces due to the height of the rod.

A Few Trigonometric Identities

- $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$
- $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
- $\sin^2\theta + \cos^2\theta = 1$
- $\sec^2\theta - \tan^2\theta = 1$
- $\csc^2\theta - \cot^2\theta = 1$

---- Extra Workspace ----



$V = \int_{5/2}^{4} 2\pi (\rho)(h)\, dy$

$4\pi \int_{5/2}^{4} (y+1)\, dy$

$4\pi \left[\frac{1}{2} y^2 + y \right]_{5/2}^{4}$