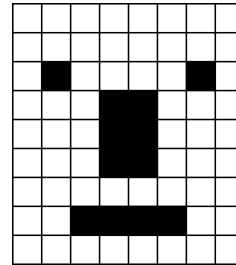


Gordon Prize Examination

February 24, 1996

1. Is $\cos(20^\circ)$ rational or irrational? Prove your answer.
2. I have an 8×9 chessboard with some spaces blacked out (as pictured). I also have a large supply of 1×3 tiles. Can the white squares be covered by 20 non-overlapping tiles?

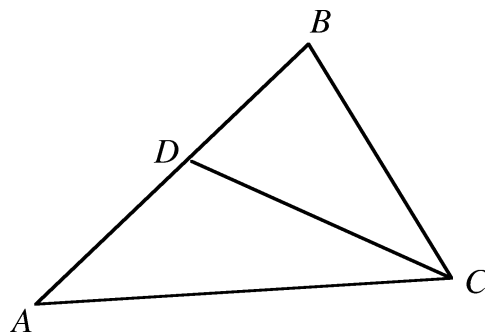


3. For $x < 0$ define

$$E(x) = \frac{1}{xe^{1/x}} \int_x^0 \frac{e^{1/s}}{s} ds.$$

Show that the integral exists, and compute the limit of $E(x)$ as x approaches 0 from below.

4. Given: \overline{CD} is the bisector of angle $\angle ACB$. Prove: $|CD| < \sqrt{|AC| \cdot |BC|}$. (Here $|CD|$ means the length of line segment \overline{CD} .)



5. Prove that there is no sequence (a_n) of positive numbers such that

$$\sum_{n=1}^{\infty} \frac{a_n}{n^2} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{a_n}$$

both converge.

6. From a calculator, it seems that $\cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} = \frac{1}{2}$. Can you prove it rigorously?

You may take this sheet with you.

Be sure to hand in separately the cover sheet

(with your name, rank, student number, and secret code name).

Put your secret code name at the top of each answer sheet.