Rasor-Bareis Prize Examination

February 21, 1998

1. A triangle has sides of lengths $a \geq b \geq c$ and area 1. Prove that $b \geq \sqrt{2}$.

2. How many real roots does the equation

$$\left|\sin x\right| = \frac{2x}{1997\pi}$$

have? [Although a picture may help you, your solution requires reasoning, not just a picture.]

3. Prove the inequality $\sin^{1998} x + \cos^{1998} x \geq \dfrac{1}{2^{998}}$.

4. Let a_n be the integer closest to $\sqrt{n}$. For example, $a_1 = 1$, $a_2 = 1$, $a_3 = 2$. Evaluate

$$\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \cdots + \frac{1}{a_{1998}}.$$

5. In how many different ways can the number 1998 be represented as a sum of two or more consecutive positive integers? (For example, 9 can be represented in two different ways: $4+5$, and $2+3+4$.)

6. Is $3\sqrt{3} - 2\sqrt{6} + 7\sqrt{5} - 5\sqrt{10}$ positive? (No calculators.)

You may take this sheet with you.
Be sure to hand in separately the cover sheet
(with your name, rank, student number, and secret code name).
Put your secret code name at the top of each answer sheet.