On Compact Support Solutions to Parabolic Problems with the p-Laplacian for $p > 2$ and Their "Counterparts" for $p < 2$

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A lecture at the Ohio State University, Columbus, OH,
Tuesday, February 23th, 2016.

Abstract

The validity of the weak and strong comparison principles for degenerate parabolic partial differential equations with the p-Laplace operator $\Delta_p(u) = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ will be discussed for $p > 2$ (the "degenerate" case) and for $1 < p < 2$ (the "singular" case). This problem is reduced to the comparison of the trivial solution ($\equiv 0$, by hypothesis) with a nontrivial nonnegative solution $u(x,t)$. The problem is closely related also to the question of uniqueness of a nonnegative solution via the weak comparison principle. In this presentation, for $p > 2$ realistic counterexamples to the uniqueness of a nonnegative solution, the weak comparison principle, and the strong maximum principle are constructed with a *nonsmooth* reaction function that satisfies neither a Lipschitz nor an Osgood standard "uniqueness" condition. Nonnegative *multi-bump* solutions with spatially disconnected compact supports and zero initial data are constructed between sub- and supersolutions that have supports of the same type. For $1 < p < 2$ we will show that a nonnegative solution $u(x,t)$ to the parabolic Cauchy problem in $\mathbb{R}^N \times (0,T)$ with nonnegative sources and nontrivial initial values $u(x,0) \geq 0$ becomes positive immediately for any $t \in (0,T_0) \subset (0,T)$, i.e., $u(x,t) > 0$ for every $x \in \mathbb{R}^N$. Finally, we adapt this result also to the Dirichlet problem in a bounded spatial domain $\Omega \subset \mathbb{R}^N$ with a completely different proof based on well-knonw results.

*Joint work with Jiří Benedikt, Peter Girg, and Lukáš Kotrla, University of West Bohemia, Czech Republic.

Keywords: Quasilinear parabolic PDE with the p-Laplacian;
nonuniqueness for the Cauchy problem;
comparison principle; sub- and supersolutions;
Barenblatt-type solutions

2000 Mathematics Subject Classification: Primary 35B05, 35B30;
Secondary 35K15, 35K55;