Catalog Description:

4580-4581 includes group theory, ring theory, vector spaces over arbitrary fields, and field theory.

Prerequisite:

C- or better in Math 4580; or credit for 5590H or 581.

Purpose:

Math 4580-4581 constitutes a two-semester sequence on abstract algebra, intended to introduce students the main concepts of this subject area. Focused on groups, rings and fields, this course gives the students a deep understanding of these three basic algebraic structures, and provides a good foundation for more specialized work. A significant goal of the course is to improve mathematical reasoning and proof writing.

Math 4581 presents special classes of groups, group actions on sets, vector spaces, and field theory that concludes with elements of Galois theory. The course places these topics in their historical context where possible.

Text:

Tom Judson, *Abstract Algebra: Theory and Applications*
https://aimath.org/textbooks/approved-textbooks/judson/

4581 Topics List (Chapters from Judson book):

12. **Matrix Groups and Symmetry**
12.1. Matrix Groups
12.2 Symmetry

13. **The Structure of Groups**
13.1. The Structure of Finite Abelian Groups
13.2. Solvable Groups

14. **Group Actions on Sets**
14.1. Group Actions
14.2. The Class Equation
14.3. Burnside's Theorem

20. **Vector Spaces** (over arbitrary fields)
20.1. Definitions and Examples
20.2. Subspaces
20.3. Independence

17. **Polynomials** (a quick 1 day review of the results we will need for field theory)

21. **Fields**
21.1. Extensions
21.2. Splitting Fields
21.3. Geometric Constructions

22. **Finite Fields** (this chapter contains both characteristic and separability; cover them first)
22.1. Finite fields

23. **Galois Theory**
23.1. Field Automorphisms
23.2. Fundamental Theorem of Galois Theory
23.3. Applications (insolvability of the quintic by radicals)