6000-Level Real Analysis Qualifying Exams Syllabus, Summer 2024

These exams are intended to test students' proficiency with the core material that is covered in Math 6211 (Real Analysis I) and Math 6212 (Real Analysis II). The main reference is Gerald B. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed., Wiley, 1999. Prerequisite material includes basic analysis at the level of Walter Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, 1976, as covered in Math 5201 (Introduction to Real Analysis I) and Math 5202 (Introduction to Real Analysis II), plus basics of set theory and metric spaces (Chapter 0 of Folland.) The distribution of material between first and second semesters (6211 and 6212) has varied from year to year. This syllabus is for the Summer 2024 Real Analysis Qualifying Exams.

Math 6211, Autumn 2023.

1 Measures
 1.1 Introduction
 1.2 σ-algebras
 1.3 Measures
 1.4 Outer Measures
 1.5 Borel Measures on the Real Line

2 Integration
 2.1 Measurable Functions
 2.2 Integration of Nonnegative Functions
 2.3 Integration of Complex Functions
 2.4 Modes of Convergence
 2.5 Product Measures
 2.6 The n-dimensional Lebesgue Integral

3 Signed Measures and Differentiation
 3.1 Signed Measures
 3.2 The Lebesgue-Radon-Nikodym Theorem
 3.3 Complex Measures
 3.4 Differentiation [of Measures] on Euclidean Space
 3.5 Functions of Bounded Variation [and Associated Measures]

4 Point Set Topology
 4.1 Topological Spaces
 4.2 Continuous Maps
 4.4 Compact Spaces
 4.5 Locally Compact Hausdorff Spaces
 4.6 Two Compactness Theorems
 4.7 The Stone-Weierstrass Theorem

5 Elements of Functional Analysis
 5.1 Normed Vector Spaces
 5.2 Linear Functionals
 5.5 Hilbert Spaces

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Math 6212, Spring 2024.

5 Elements of Functional Analysis
  5.3 The Baire Category Theorem and its Consequences

6 L^p Spaces
  6.1 Basic Theory of L^p Spaces
  6.2 The Dual of L^p
  6.3 Some Useful Inequalities

8 Elements of Fourier Analysis
  8.1 Preliminaries
  8.2 Convolutions
  8.3 The Fourier Transform
  8.4 Summation of Fourier Integrals and Series
  8.5 Pointwise Convergence of Fourier Series
  8.7 Applications to Partial Differential Equations

9 Elements of Distribution Theory
  9.1 Distributions
  9.2 Compactly Supported Distributions and Tempered Distributions