2025 Rasor-Bareis examination problems

1. Prove that $n = 1\underbrace{00\ldots00}_{2024}1$ is not prime.

2. Evaluate $\displaystyle\int_{-1}^{1} \frac{dx}{(e^x+1)(x^2+1)}$.

3. Prove that for every positive integer n, the number $\sqrt[n]{\sqrt{3}+\sqrt{2}} + \sqrt[n]{\sqrt{3}-\sqrt{2}}$ is irrational.

4. Let $f\colon (0,\infty) \longrightarrow (0,\infty)$ be an increasing function (meaning that $x < y$ imlpies $f(x) \le f(y)$) satisfying $\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)} = 1$. Prove that $\lim\limits_{x\to\infty} \frac{f(cx)}{f(x)} = 1$ for any $c > 0$.

5. The points of $\mathbb{R}^2$ are colored in two colors. Prove that there exists a triangle whose sides have lengths $1, \sqrt{3}, 2$ and whose vertices have the same color.

6. Let P be an equiangular polygon (meaning that all the angles of P are equal) and let x be a point inside P. Prove that the sum of the distances from x to the lines containing the sides of P doesn't depend on the choice of x.