Can One Hear the Shape of a Drum?

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Background

The problem "Can One Hear the Shape of a Drum" was firstly brought up by Mark Kac in 1966 and became famous very soon. The answer to the question is no, solved by Carolyn Gordon, David Webb, and Scott Wolpert in 1992.

The question of whether one can hear the shape of a drum is of mathematical interest because it has a clean mathematical formulation in terms of the Laplacian operator, an operator with many uses in mathematics and physics. Specifically, if we accept an idealized mathematical model of a drum, the question reduces to “Do the eigenvalues of the Laplacian operator associated to a 2D shape completely determine the geometry of the shape?”

Hearing is the ear detecting changes in the air. Specifically, it is the change in the ear hairs' position or direction due to the air. Ear hair moves in response to shockwaves of air, commonly known as sound. These ear hairs can detect differences in the rate that shockwaves happen. If two drums make the air behave the same exact way, producing the shockwaves at the same rate, they would sound the same.

Drums change the air by making their membrane move up and down, pushing and “pulling” air particles. The movement of the membrane going up and down is a result of the tension attempting to reach an equilibrium. The tension force is larger the further the membrane is from equilibrium.

Design

For our final design, we 3D-printed two pairs of $\frac{1}{2}$” thick discs with our desired shapes cut out of the center, stretched a latex balloon between them to mimic a drum head, and held our drums together with clamps.

Other ideas that we considered but eventually rejected include the following:

- Magnets
  - We wanted to hold our discs together with neodymium magnets but realized that they can be expensive
- Wood
  - At one point, we were going to hold our drum head together with two pieces of wood with our shapes cut out of them. Upon doing some research into the process, we realized we weren't certified with the equipment necessary to make this happen.
- Metal
  - Metal was one of our choices for our drum head, but we realized that it was very different from the drum head mathematically. The differential equation that governs the vibration of a metal plate is significantly different from the wave equation used in the mathematical model of a drum.
- Mylar
  - Was going to use this for our drum head but found a cheaper solution
- Rubber
  - Too difficult to stretch to become a drum head
- Latex Glove
  - Wasn't large enough to fit around the embroidery hoop that we were using as a stand in for our later designs with clamps
- Trash Bags and T-Shirts
  - Weren't taut enough to be drum heads

Experiments

To test the drums, we took careful recordings of both drums using a microphone with high sample rate and bit depth. If the drums conformed closely enough to the simple mathematical model, then the frequency content of their sounds should be very similar.

To visualize the underlying frequencies in our three drums, we used the program Wave Candy to produce spectrograms for both recordings. The spectrograms are shown to the right. The horizontal axis represents time, and the vertical axis represents frequency. Each horizontal streaks come from a frequency peak that persisted after striking a drum. Each of the two patterns of streaks comes from striking one of the two drums.

As you can see, the spectrograms for the two drums are somewhat similar, but not identical. We expect this is because of the way we stretch the balloon over the disc when forming the drum; the mathematical model assumes the drum head is under uniform tension in all directions, but our method for stretching the balloons over the discs is somewhat haphazard. We expect we could get the spectrograms to more closely align by devising a more precise method for stretching the balloons.