Seifert Surfaces of Knots

Killian Davis Alison Kraniske

Department of Mathematics

Introduction

Knot theory is a branch of mathematics that studies the topological, algebraic, and combinatorial properties of knots. It has found applications in molecular biology, quantum physics, and material science. For this project, we built a Seifert surface using glass blowing techniques.

Seifert Surfaces

A Seifert surface is a compact, connected, oriented surface whose oriented boundary is a knot. Every knot has a Seifert surface, which can be found using the Seifert algorithm. For the Seifert algorithm [1]:

1. Smooth each crossing with respect to orientation. This results in disks.
2. Place each disk at different heights.
3. Connect the disks with half twists, respecting orientation.

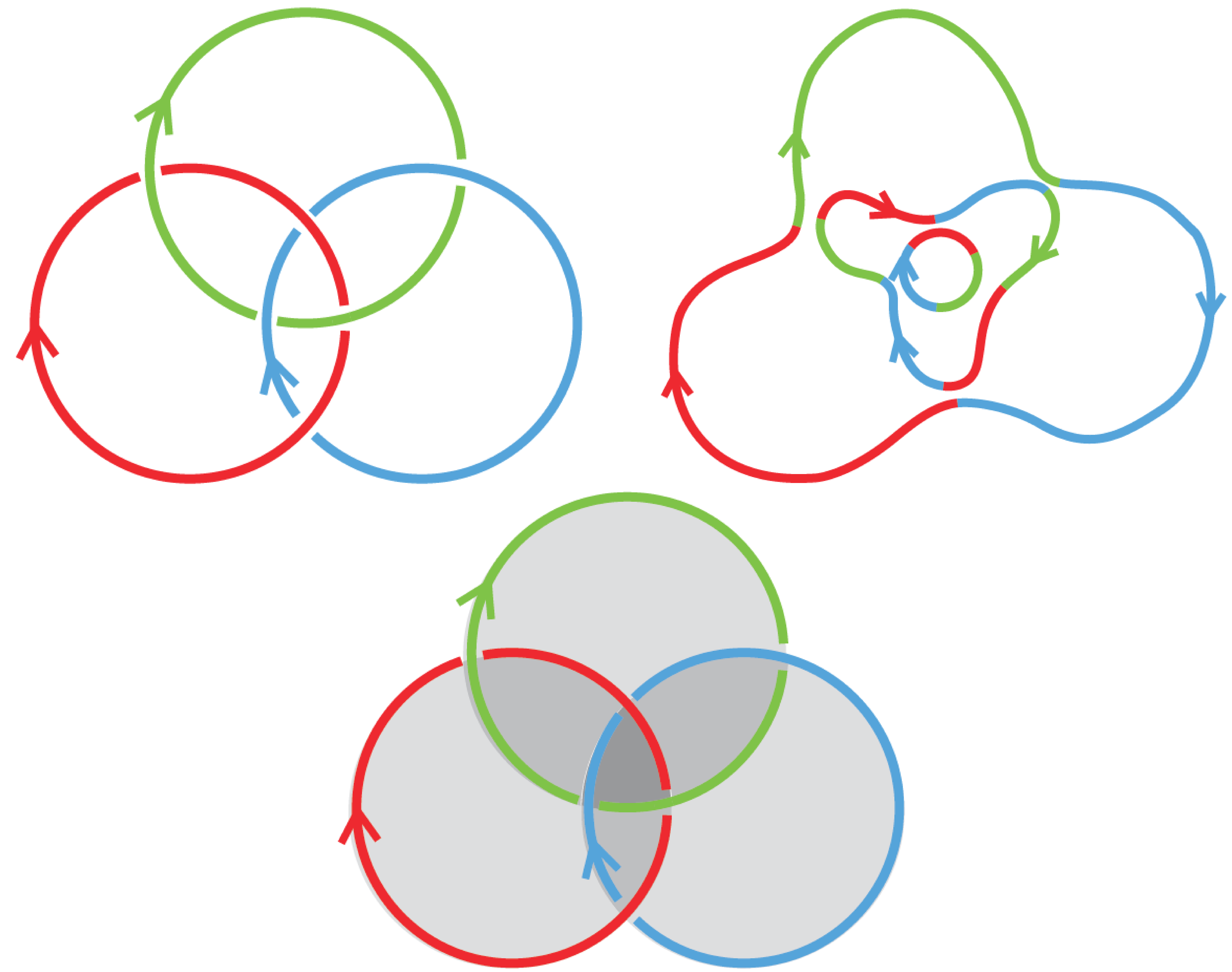
Genus of a Knot

The genus of a knot is the minimal genus of any Seifert surface for a given knot. The genus respects knot addition [2]:

$$g(K_1) + g(K_2) = g(K_1 + K_2)$$

This implies that knots do not have additive inverses, genus one knots are prime, and knots decompose into prime knots.

A Seifert Surface of the Borromean Rings



Steps 1, 2, and 3 of the Seifert algorithm

Glass Construction Process

In order to construct a Seifert Surface out of glass, I had to twist sheet glass in the hotshop. The sculpture is constructed out of COE 96 sheet so that it is compatible with our furnace glass. I brought up the strips in a kiln to 900 degrees so that they would not go through thermal shock, then picked them up on the end up of a metal rod with a bit of hot glass called a punty. I then introduced the glass to the reheating chamber, which is kept at 2,000 degrees, to get the sheet to working temperature. I then spot heated the center of the strip with an oxy-propane torch, grabbed the end of the sheet, and twisted my punty to get the 180 twist. After annealing the strips, I glued the structure together with epoxy adhesive.

Mathematics and Art

This project seeks to explore mathematics and art as mutually beneficial disciplines. In the past century of modern art, form in painting and structure of sculpture often mimic beauty inherent in the visuals and aesthetics of mathematics. Knots in particular have been widely used as decorative elements since manuscripts in the Middle Ages. Putting knots in both a contemporary art context through the medium of glass and a mathematical context through the accurate construction of a Seifert Surface allows us to more deeply consider the cross-discipline connections between what is beautiful, innovative, and practical. Part of the goal of this work is to create art objects that make concepts of higher math more accessible and tangible, challenging the adversity that many artists cite from their experience in math. In turn, the work highlights the necessary creativity and dedication of mathematicians to their discipline, the same as an artist is devoted to the medium. The clarity, fragility, and technical difficulty of the glass mathematical model is a stark contrast from the paper and plaster mathematical forms created in the early nineteenth century. The material itself is illustrative and compliments the orientation of the Seifert Surface as the entire object can be seen at once. Without the mathematical context, the work stands as a sculpture that explores balance, stability, interconnection, continuity, and pushing the materiality of glass.

References

[1] Colin Adams. *The Knot Book: An Elementary Introduction to the Mathematical Theory of Knots*. American Mathematical Society, 2004.

[2] W. B. Raymond Lickorish. *An Introduction to Knot Theory*. Springer, 2012.