DEPARTMENT OF MATHEMATICS

Introduction

Sudoku is a number puzzle game where the goal is to fill a grid with numbers according to some set of rules. Typically, the rule is to fill a 9×9 grid with the numbers 1-9 such that in every row, column, and in every one of nine 3×3 blocks, each of the numbers from 1 to 9 appear only once.

For humans, we typically have to consider all possible numbers that could go into a square, and reduce the possibilities to one single number. While anyone who has learned Sudoku knows some basic techniques to eliminate possibilities, there exists more esoteric techniques as well.

In our research we analyze the number and kinds of puzzles solvable by human techniques.

Definitions

- A house is the group of nine cells contained in one row, column, or box.
- Pencil marks are small numbers written inside each cell that represent the possible values that cell could hold.
- A naked single is a cell where, after eliminating based on given board, there is only one pencil mark left.
- A hidden single is a cell which contains a pencil mark that exists no where else in any house it is contained in.
- A locked candidate is a pencil mark that, within a box, only exists in one row (column), precluding it from existing in that row (column) outside the box.

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Example Puzzle

6	1	9	$\begin{array}{c}2\\4\\8\end{array}$	2 4 7	4 7	4 8	3	$\begin{array}{c}4&5\\&8\end{array}$
3	3 8	2	5	3 4	9	7	1	$\begin{array}{cc} 4 & 6 \\ & 8 \end{array}$
3 5	4	7	3 8	6	1	2	5 8 9	5 8 9
$egin{array}{cccc} 1 \\ 4 & 5 \end{array}$	5	8	$\frac{3}{4}$	$\begin{array}{c} 3\\4\\9\end{array}$	3 4	6	2	7
4	2 6	4 6	7	1	5	9	4 8	$\begin{array}{c} 3\\4\\8\end{array}$
7	9	3	$\begin{smallmatrix}&2\\4&6\end{smallmatrix}$	8	4 6	$\frac{1}{4}$	4 5	4 5
2	3 6	5	9	$\begin{array}{c} 3\\4\\7\end{array}$	8	$\frac{3}{4}$	4 6	1
8	7	4 6	1	3 4	2	5	$\begin{array}{cc} 4 & 6 \\ & 9 \end{array}$	$\begin{smallmatrix}&&3\\&&6\\&&9\end{smallmatrix}$
9	3 6	$\begin{array}{ccc} 1 \\ 4 & 6 \end{array}$	$\begin{array}{c} 3\\ 4 & 6\end{array}$	$\begin{array}{c} 3\\ 4 5\end{array}$	$\begin{array}{c} 3\\ 4 \end{array} 6$	$\begin{array}{c} 3\\4\\8\end{array}$	7	2

Example Algorithm

procedure NAKED_TUPLES(*board*, *n*) for *House* in {*rows*, *columns*, *boxes*} do $C \leftarrow \text{set of cells in house}$ remove from C all cells $\geq n$ pencil marks for each $X \in \binom{C}{< n}$ (combinations $\leq n$ cells in C) do $naked_tuple \leftarrow \bigcup$ all pencil-marks of cells in x if $|naked_tuple| = n$ then Erase all of the marks from *naked_tuple* from... ...every other cell in house end if end for

end for end procedure

References

- [1] Blagovest Dachev. sudoku (soduku puzzle generator software). https://github.com/dachev/sudoku,2010. Accessed: March 27, 2025.
- [2] Bernhard Hobiger. Hodoku: Human style solving techniques for sudoku. https://hodoku.sourceforge.net/en/techniques.php, 2008. Accessed: February 12, 2025.
- [3] Takayuki Yato and Takahiro Seta. Complexity and completeness of finding another solution and its application to puzzles. IEICE Transactions on Fundamentals of EECS, E86-A, 05 2003.

6	1	9	8	2	7	4	3	5
3	8	2	5	4	9	7	1	6
5	4	7	3	6	1	2	9	8
1	5	8	4	9	3	6	2	7
4	2	6	7	1	5	9	8	3
7	9	3	2	8	6	1	5	4
2	6	5	9	7	8	3	4	1
8	7	4	1	3	2	5	6	9
9	3	1	6	5	4	8	7	2

If a 3×3 puzzle is solvable by human techniques, than the solution must be unique. Furthermore, if a solution can be found by human techniques, it can be found in at most 648 steps.

We made some initial observations about select human techniques and formulated a lemma about when we can solve a puzzle. Informed by this, we limited our research to puzzles with a unique solution.

Running through 50,000 generated puzzles [1], using 3 human techniques, our algorithm was able to solve 52% of them. On average, it filled about 38 cells and "erased" 415 pencil marks.

Also, looking at 50 infamously difficult 17-clue solutions, we were able to solve 40 of them using these human techniques alone.

Question: How effective will the human techniques be as we scale to larger Sudoku boards?

Question: We solved a 17-clue Sudoku using human techniques. This is the least clues possible in the 3×3 case. What is the best we can do for larger Sudoku grids?

Question: What is the *most* filled puzzle with a unique solution that human techniques cannot solve?

Future Goal: Develop theories about what techniques are used with what frequency.



Lemma

Results

Future Directions