2024 Algebra 6111 Qualifying Exam

Instructions

Enter your name.# on the roster sheet together with a code name for yourself that is different from any code name that has already been entered.

Answer each question on a separate sheet of paper, and write your code name and the problem number on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.

Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment that you use in this respect is an important part of the exam.

This is a closed book, closed notes exam.

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1. Let G be a finite simple group and H a proper subgroup of G of index $n > 1$. Prove that if a prime p divides $|G|$, then $p \leq n$.

2. Prove that a finite abelian subgroup in $\mathrm{SL}_2(\mathbb{C})$ must be cyclic.

3. Let X be an $n \times n$ matrix over $\mathbb{Q}$. Show that X is nilpotent if, and only if, $\mathrm{Tr}(X^k) = 0$ for all $k \geq 1$.

4. Define the ideal $I = (x, y, z)$ in $\mathbb{C}[x, y, z]$. Show that I cannot be generated by fewer than three elements in $\mathbb{C}[x, y, z]$.

5. Let R be a commutative ring. Let $S \subset R$ be a multiplicatively closed set containing 1, but $0 \notin S$. Let $\mathfrak{p}$ be a maximal element in the set of ideals of R whose intersection with S is empty. Show that $\mathfrak{p}$ is prime in R and $S^{-1}\mathfrak{p}$ is prime in $S^{-1}R$.