

2026 Gordon examination problems

1. Let p be a nonconstant polynomial with integer coefficients; prove that the number $\overline{0.p(1)p(2)\dots}$ (the concatenation of the decimal expansions of the integers $p(1), p(2), \dots$) is irrational.
2. A point (n, m) of \mathbb{Z}^2 is said to be primitive if n and m are coprime integers. Prove that there are arbitrarily large disks in \mathbb{R}^2 containing no primitive points.
3. Suppose the polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + 1$ with $a_1, \dots, a_{n-1} \geq 0$ has n real roots. Prove that $p(2) \geq 3^n$.
4. Prove that
$$\int_{-\pi}^{\pi} \frac{\sin(nx)}{(1+2^x)\sin x} dx = \begin{cases} 0, & n \text{ is even} \\ \pi, & n \text{ is odd.} \end{cases}$$
5. Suppose that each point of \mathbb{R}^3 is colored with one of 3 colors. Prove that the pairs of points in at least one of these colors achieve all positive distances.
6. Prove that any nonzero real $n \times n$ -matrix A with $n \geq 2$ can be represented as $A = B + C$ with $\det B = \det C = 1$.