2026 Rasor-Bareis examination problems

1. Prove that the number $0.1234567891011121314\ldots$ is irrational.

2. Let $n \geq 4$ and let $a_1, \ldots, a_n$ be positive real numbers with $a_1 \cdots a_n = 1$. Prove that

$$\frac{1}{1+a_1+a_1a_2} + \frac{1}{1+a_2+a_2a_3} + \cdots + \frac{1}{1+a_n+a_na_1} > 1.$$

3. Find the limit of the sequence of the fractional parts of $(2+\sqrt{2})^n$, $n = 1, 2, \ldots$

4. Given a triangle T, prove that for any 2-coloring of the points of the plane there are three points of the same color that form a triangle similar to T.

5. Prove that $\int_0^3 \sqrt[4]{x^4+1}\,dx + \int_1^3 \sqrt[4]{x^4-1}\,dx > 9$.

6. Find all functions $f\colon \mathbb{R} \setminus \{0,1\} \longrightarrow \mathbb{R}$ satisfying $f(1/x) + f(1-x) = x$ for all $x \neq 0, 1$.