

2026 Algebra 6112 Qualifying Exam
Instructions

Enter your name.# on the roster sheet together with a code name for yourself that is different from any code name that has already been entered.

Answer each question on a separate sheet of paper, and write your code name and the problem number on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.

Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment that you use in this respect is an important part of the exam.

This is a closed book, closed notes exam.

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1. Let R be a commutative ring. Let $(I, \leq)$ be a directed set, and let $\mathcal{N} = (N_i, f_{ij})$, $\mathcal{M} = (M_i, g_{ij})$, and $\mathcal{P} = (P_i, h_{ij})$ be directed systems of R -modules over I (the notation means, for instance, that for $i \leq j$, $f_{ij}: N_i \rightarrow N_j$ is the given map in the directed system). Suppose we are given an exact sequence of directed systems $\mathcal{N} \xrightarrow{a} \mathcal{M} \xrightarrow{b} \mathcal{P}$, i.e. for all $i \in I$ we have an exact sequence of R -modules

$$N_i \xrightarrow{a_i} M_i \xrightarrow{b_i} P_i$$

compatible with the transition maps in the directed systems ($g_{ij}a_i = a_jf_{ij}$ and $h_{ij}b_i = b_jg_{ij}$ for all $i \leq j$). Show that the induced sequence of direct limits

$$\varinjlim N_i \rightarrow \varinjlim M_i \rightarrow \varinjlim P_i$$

is exact.

2. Prove that for all abelian groups A and B and all $n > 1$, $\text{Ext}_{\mathbb{Z}}^n(A, B) = 0$.
3. Show that $F = \mathbb{Q}(\sqrt{2 + \sqrt{2}})$ is Galois over $\mathbb{Q}$, and compute $\text{Gal}(F/\mathbb{Q})$.
4. Compute the number of monic irreducible polynomials of degree 3 over $\mathbb{F}_5$.
5. Let F (for simplicity) be an infinite field, and let K/F be a separable field extension of finite degree. Prove that $K = F(\alpha)$ for some $\alpha \in K$ (the theorem of the primitive element).