Syllabus for Core Algebra Sequence Math 6111–6112
& Algebra Qualifying Exams

Math 6111 covers the basic concepts of algebra that are that are necessary for graduate level core mathematics. This includes groups, rings, modules, and linear & multilinear algebra. Math 6112 covers more advanced topics in algebra that are necessary for those planning to work in areas that require more algebra, such as algebraic geometry, algebraic topology, commutative algebra, representation theory, and number theory. The topics include category theory, homological algebra, and Galois theory. There are two algebra qualifying exams, one covering the material in Math 6111 and the other the material in Math 6112.

Basic References: Most of the material for Math 6111 and 6112 can be found in the following books:

1. Serge Lang, *Algebra* (3rd edition, Springer)
2. Nathan Jacobson, *Basic Algebra I & II* (2nd edition, Dover)

Secondary References: Additional references for special topics include:

3. M. Atiyah and I.G. Macdonald, *Introduction to Commutative Algebra* (Addison-Wesley)
4. Emil Artin, *Galois Theory* (Dover and/or AMS)

Math 6111

Group Theory:

- Monoids, groups, homomorphisms, subgroups, normal subgroups and quotient groups. Groups acting on sets. Counting lemmas.
- Basic isomorphism theorems. Semi-direct products. Short exact sequences.
- Sylow theorems.
- Composition series. Jordan–Hölder series. Solvable groups and nilpotent groups.
- Simple groups. Simplicity of the alternating groups.

Lang: I.1–5
Jacobson I: 1.1–10, 1.12; 4.6
Jacobson II: 3.3

Rings and Modules:

- Basics of ring theory: definitions of rings, subrings, ideals, modules. Ring homomorphisms.
- Integral domains. Characteristic of a ring. Chinese remainder theorem.
- Prime and maximal ideals. Prime avoidance. Rings and modules of fractions. Localization.
- Noetherian rings and Hilbert basis theorem. Primary decomposition.
- UFD and PID. Polynomials rings over UFD's. Irreducibility criterion. Symmetric polynomials. Discriminant.
- Modules over PID's.

Lang: II.1–3, II.5; IV.1–4, IV.6
Jacobson I: 2.1–7, 2.10–11, 2.13–16
Jacobson II: 7.1–4, 7.9
Atiyah-Macdonald: 1, 2, 3, 7

Linear and Multilinear Algebra:

- Vector spaces. Basic operations on vector spaces: direct sum, tensor product, dual vector spaces.
- Bilinear and multilinear forms. Quadratic forms. Positive definite and semidefinite forms.
- Tensor algebra. Symmetric and Exterior algebra.[1] Definition of determinant and minors of an endomorphism.
- Automorphisms preserving a bilinear form: symplectic and orthogonal matrices.
- Polar, Gauss and Jordan decompositions of a matrix.

Lang: XIII, 1–6; XIV. 1–3; XV. 1–4, 8;XVI,1, 2, 7, 8; XIX.1
Jacobson I: 3.10; 6.1–6.9; 7.1–7.2
Jacobson II: 3.9

[1] An additional reference for multilinear algebra: S. S. Chern, W. H. Chen and K. S. Lam *Lecture on differential geometry, Ch. 2*, World Scientific.

Category Theory:

- Categories, dual categories, universal objects.
- Covariant and contravariant functors, representable functors, natural transformations.
- Universal objects, products and coproducts.
- Inverse limits and direct limits.

Lang: I.10–12, III.10
Jacobson II: 1.1–8, 2.5, 2.9

Homological Algebra:

- Modules, homomorphisms, the *Hom* functors.
- Direct products and direct sums of modules. Abelian categories.
- Free, projective and injective modules. Tensor products.
- The snake lemma, complexes, homology sequence.
- Projective and injective resolutions. Derived functors, *Ext* and *Tor*.

Lang: III.1–4; XX.1–2, XX.4–6
Jacobson II: 3.1, 6.1–8, 7, 10, 11

Fields and Galois Theory:

- Algebraic extensions, algebraic closure.
- Splitting fields, normal extensions. Separable and inseparable extensions. Simple extensions.
- Finite fields, perfect fields.
- Independence of characters.
- Galois theory for finite extensions.
- Cyclotomic extensions, Kummer extensions, radical extensions, and solvable extensions.

- Galois theory for infinite extensions and the Krull topology.
- Transcendental extensions.
- The general equation of n^{th} degree, Abel's Theorem, Galois groups of polynomials with integral coefficients.

Lang: V.1–6, VI.1–8, VI.14, VIII.1;
Jacobson I: 4.1, 4.4–9, 4.13–14;
Jacobson II: 8.1–2, 8.6–9, 8.12–13
Artin

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