6000-Level Analysis Qualifying Exams Syllabus, Summer 2020

These exams are intended to test students' proficiency with material that covered in Math 6211 (Real Analysis I) and Math 6212 (Real Analysis II). The distribution of material between first and second semesters (6211 and 6212) has varied from year to year. The main reference is Gerald B. Folland, *Real Analysis: Modern Techniques and Their Applications, 2nd ed*., Wiley, 1999. Prerequisite material includes basic analysis at the level of Walter Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, 1976, as covered in Math 5201 (Introduction to Real Analysis I) and Math 5202 (Introduction to Real Analysis II), plus basics of set theory and metric spaces (Chapter 0 of Folland's text). This syllabus applies for Summer 2020.

Math 6211, Autumn 2019.

Chapter 1 Measures
- 1.1 Introduction
- 1.2 σ-algebras
- 1.3 Measures
- 1.4 Outer Measures
- 1.5 Borel Measures on the Real Line

Chapter 2 Integration
- 2.1 Measurable Functions
- 2.2 Integration of Nonnegative Functions
- 2.3 Integration of Complex Functions
- 2.4 Modes of Convergence
- 2.5 Product Measures
- 2.6 The n-dimensional Lebesgue Integral

Chapter 3 Signed Measures and Differentiation
- 3.1 Signed Measures
- 3.2 The Lebesgue-Radon-Nikodym Theorem
- 3.3 Complex Measures
- 3.4 Differentiation of Measures on Euclidean Space
- 3.5 Functions of Bounded Variation

Chapter 4 Point Set Topology
- 4.1 Topological Spaces
- 4.2 Continuous Maps
- 4.3 Nets
- 4.4 Compact Spaces
- 4.5 Locally Compact Hausdorff Spaces
- 4.6 Two Compactness Theorems
- 4.7 The Stone-Weierstrass Theorem

Chapter 5 Elements of Functional Analysis
- 5.1 Normed Vector Spaces
- 5.2 Linear Functionals
- 5.3 The Baire Category Theorem and its Consequences
- 5.4 Topological Vector Spaces
- 5.5 Hilbert Spaces

Chapter 7 Radon Measures
- 7.1 Positive Linear Functionals
- 7.2 Regularity and Approximation Theorems
- 7.3 The Dual of $C_0(X)$
- 7.4 Products of Radon Measures

Math 6212, Spring 2020.

Chapter 6 L_p Spaces
- 6.1 Basic Theory of L_p Spaces
- 6.2 The Dual of L_p
- 6.3 Some Useful Inequalities
- 6.4 Distribution Functions and Weak L_p
- 6.5 Interpolation of L_p Spaces

Chapter 8 Elements of Fourier Analysis
- 8.1 Preliminaries
- 8.2 Convolutions
- 8.3 The Fourier Transform
- 8.4 Summation of Fourier Integrals and Series
- 8.5 Pointwise Convergence of Fourier Series
- 8.7 Applications to Partial Differential Equations

Chapter 9 Elements of Distribution Theory
- 9.1 Distributions
- 9.2 Compactly Supported, Tempered, and Periodic Distributions
- 9.3 Sobolev Spaces