Ph.D. Algebra Qualifying Exam
6111

August 17, 2017
Proctor: Angelica Cueto and/or Sachin Gautam

Directions

1. Initial the roster sheet and enter a code name for yourself that is different from any code name that has already been entered.
2. Answer each question on a separate sheet or sheets of paper, and write your *code name* and the *problem number* on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.
3. Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment you use in this respect is an important part of the exam.
4. This is a two hour, closed book, closed notes exam.

1. Show that any group of order 360 must have a subgroup of order 10.

2. Let G be a group and K a normal subgroup of G. Show that G has a composition series if and only if both K and G/K have composition series.

3. Let G be a finite group of order n and suppose its center C has order c. Assume ρ is an irreducible complex representation of G with dimension d. Show $d \leq \sqrt{n/c}$.

4. Let R be a commutative ring, $\mathfrak{p}_1, \ldots, \mathfrak{p}_n$ prime ideals of R. Let $S = \cap_{i=1}^{n}(R - \mathfrak{p}_i)$. Show that any prime ideal of $S^{-1}R$ has the form $S^{-1}\mathfrak{p}$ where $\mathfrak{p}$ is a prime ideal of R contained in one of the $\mathfrak{p}_i$.

5. Show that if $\mathfrak{a}$ is an ideal in a commutative ring R with the property that $\mathfrak{a}$ is not finitely generated but every ideal properly containing $\mathfrak{a}$ is finitely generated then in fact $\mathfrak{a}$ is a prime ideal. Use this to show that R is Noetherian iff every *prime* ideal of R is finitely generated.