Ph.D. Algebra Qualifying Exam
6111

August, 2015
Proctor: Jim Cogdell

Directions

1. Initial the roster sheet and enter a code name for yourself that is different from any code name that has already been entered.

2. Answer each question on a separate sheet or sheets of paper, and write your *code name* and the *problem number* on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.

3. Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment you use in this respect is an important part of the exam.

4. This is a closed book, closed notes exam.

1. Let G be a finite group of order 216. Show that G is not simple.

2. Let G be a group, and let $G^{(n)}$ denote the n-th term in the derived series for G. Prove that G is solvable if and only if $G^{(n)} = 1$ for some $n \geq 0$.

3. Let G be a finite group of order pq with p and q primes such that $p < q$ and $q \not\equiv 1 \pmod{p}$. Use representation theory to show that every irreducible representation of G is one dimensional and hence that G is abelian.

4. Let R be a commutative Noetherian ring with 1. If $S \subset R$ a subring, must S be Noetherian? If $Q = R/I$ is a quotient ring, must Q be Noetherian? (In each case, prove or give s counter example.)

5. Let R be a commutative ring with 1 and let S be a multiplicatively closed subset of R such that $1 \in S$.

   (a) Describe the prime ideals in the ring of fractions $S^{-1}R$.

   (b) Let $S = \{f^n\}_{n \geq 0}$ where $f \in R$ is not nilpotent. Use (a) to show that there exists a prime ideal P such that $f \notin P$.