# Lie Groups and Their Representations

## **Instructor and Class Information**

Lecturer:	Course Num.:
Office:	Lecture Room:
Phone:	Lecture Times:
Email:	Office Hours:

## **About Course Goals**

#### FORMAT

The course will meet three times a week for 55 minutes each meeting. Instructions will be mainly by lecture delivered by the instructor. It may also include occasional in-class discussion as well as short student presentations, particularly, by post-candidacy students.

#### **DESCRIPTION & GOALS**

This course provides students with a solid working knowledge in the theory of Lie groups and their representations. We will primarily focus on analytic and algebraic aspects of compact Lie groups through a wide variety of examples. These objects themselves and techniques used to study them are prevalent in several areas of mathematics. This course should be of interest to students working in or planning to work in representation theory, number theory or algebraic geometry.

#### PREREQUISITES

Math 7161 and 6701.

#### Textbook

#### MAIN REFERENCE

#### Instructor Lecture Notes

W. Rossmann: "Lie Groups: an introduction through linear groups" Oxford Graduate Texts in Mathematics 2002. ISBN: 9780198596837.

#### **ADDITIONAL REFERENCES**

Lawrence Conlon: "Differentiable Manifolds". Birkhäuser Boston, 2001. ISBN:0817641343.

A. Knapp: "Lie Groups beyond an introduction". Birkhäuser Boston, 2002.

C. Chevalley: *"Theory of Lie Groups"* Princeton Landmarks in Mathematics ISBN: 9780691049908 *Manifolds"*. Springer, 2002. ISBN:0387954481.

#### Assessments

#### **HOMEWORK ASSIGNMENTS**

There will be approximately 10 homework assignment sheets, which will typically contain several fully described problems as well as a list of numbers of textbook problems. Due dates of

assignments will announced and set typically a week after the assignments are published

#### **FINAL PROJECT**

Besides homework assignments, students' grade will be based on a final in-class short presentation. A list of suggested topics will be published about three to four weeks before the end of classes and student presentations will be organized during the finals week.

#### **CLASS PARTICIPATION AND ATTENDANCE**

Although attendance is not regularly monitored frequent absences are likely to be noted and may factor into the grade in borderline cases.

## Grading

#### **COURSE SCORE**

A course score will be computed from the above assessments. Homework assignments will count 70% towards the grade and the final presentation 30%.

## LETTER GRADES

Letter grades will be determined based on the course score. The approximate minimum scores letter grades are 80% for an "A", 73% for an "A-", 67% for a "B+", 55% for a "B-", and 40% for a "C-". The exact cut-off scores may vary depending on the difficulty of assignments.

## **Weekly Schedule**

Week 1	Definitions and Examples: Lie Groups, Differential Forms, Vector Fields, Lie algebras	
Week 2	Correspondence between Lie groups and Lie algebras. Exponential map. Adjoint representation	
Week 3	Semisimple and Unipotent elements. Jordan decomposition. Solvable and nilpotent Lie groups. Lie's theorem.	
Week 4	Compact Lie groups. Maximal tori. Weyl group.	
Week 5	Compact Lie groups cntd. Root systems.	
Week 6	Classical (linear) Lie groups. Connectedness and simply-connectedness.	
Week 7	Classification of root systems.	
Week 8	Complex semisimple Lie groups and algebras. Cartan-Killing form. Compact real form of a complex Lie group.	
Week 9	Integration on compact Lie groups. Weyl's integration formula.	
Week 10	Representations. Complete reducibility theorem. Orthogonality of irreducible characters.	
Week 11	Peter-Weyl theorem. Classification of irreducible representations.	
Week 12	Weyl's character formula and its applications.	
Week 13	Homogeneous spaces. Construction of irreducible representations: Borel-Weil-Bott theorem.	
Week 14	Decomposition of tensor product of irreducible representations. Combinatorial formulae for tensor product multiplicities.	

# **General Policies**

## ACADEMIC MISCONDUCT

It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term "academic misconduct" includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-487). For additional information, see the Code of Student Conduct (http://studentaffairs.osu.edu/info\_for\_students/csc.asp)."

# **DISABILITY SERVICES**

Students with disabilities that have been certified by the Office for Disability Services will be appropriately accommodated and should inform the instructor as soon as possible of their needs. The Office for Disability Services is located in 150 Pomerene Hall, 1760 Neil Avenue; telephone 292-3307, TDD 292-0901; http://www.ods.ohio-state.edu/.