## **Differences Between Various Classes of Module Hulls**

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While the notions of Baer, quasi-Baer, and extending modules are related to each other, it is well known that these are distinct notions in general. We discuss a number of instances under which these notions coincide with each other and other instances when these are distinct and disparate from each other. As a consequence, for a certain module M over a commutative ring and a fixed injective hull E(M), we study the coincidence, similarities and contrasts between Baer module hull, the quasi-Baer module hull, and the extending module hull of M.

We show that for an essential extension  $V_R$  of a nonsingular cyclic module over a commutative ring R, the notions of  $V_R$  being Baer, quasi-Baer, quasicontinuous, extending, and FI-extending, or strongly FI-extending, coincide with each other. Using this rsult, we prove the existence and coincidence of the Baer, the quasi-Baer, the extending hulls of any nonsingular cyclic module over a commutative ring. As application, such module hulls are explicitly described. As a byproduct, when R is a commutative semiprime ring, all intermediate Baer (extending, quasi-continuous) ring between R and Q(R), where Q(R) is the maximal (right) ring of quotients of R.

A characterization of a commutative domain to be Prüfer is obtained by using extending module hulls of certain modules. A number of examples which exhibit the disparities and differences between Baer hulls, quasi-Baer hulls, extending hulls, and FI-extending hulls of a module are provided.

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