Instructions:

• You have **1 hour and 45 minutes** to complete this exam. It consists of 12 problems on 13 pages including this cover sheet and is worth a total of 200 points. The value of each question is listed below and with each question.

• You may not use any books or notes during this exam.

• Calculators are permitted EXCEPT those calculators that have symbolic algebra or calculus capabilities. In particular, the following calculators and their upgrades are not permitted: TI-89, TI-92, and HP-49. In addition, neither PDAs, laptops nor cell phones are permitted.

• Make sure to read each question carefully.

• Please **write clearly** and make sure to **justify your answers**. Correct answers with no supporting work may receive no credit. Unless otherwise stated, solutions found by graphing will receive no credit.

• Please write your answers on the indicated lines.

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(1). Find the following limits:

(a) (3 points) \( \lim_{x \to 9} \frac{5x^2 + 9x + 5}{x^2 - 18x + 81} = \) 

(b) (3 points) \( \lim_{x \to 4^+} \frac{6x^2 - 13x - 63}{x^2 - 2x - 8} = \) 

(c) (3 points) \( \lim_{x \to -6} \frac{3x^2 + 16x - 12}{x^2 + 2x - 24} = \) 

(d) (3 points) \( \lim_{x \to \infty} \frac{3x^2 - 7x^5 + 4x^3}{5x^5 + 9x^3 - 3x^2} = \)
(2). Given

\[
f(x) = \begin{cases} 
\frac{5 - x}{x^2 - 3x - 10} & \text{if } x \leq 5 \\
\frac{x - 5}{x^2 - 17x + 60} & \text{if } x > 5
\end{cases}
\]

(a) (2 points) \( \lim_{x \to 5^-} f(x) = \) __________

(b) (2 points) \( \lim_{x \to 5^+} f(x) = \) __________

(c) (4 points) Find all points of discontinuity for \( f(x) \) __________

(3). Given \( y = e^{4x^2 - 5x - 75} + 8 \)

(a) (8 points) Find the slope of the tangent line to the graph of this equation when \( x = 5 \).

Answer (3a): slope = ________________

(b) (3 points) Find an equation of the tangent line to the graph of this equation when \( x = 5 \).

Answer (3b): \( y = \) ________________
(4). (14 points) Use the definition of the derivative given below to find $f'(x)$ where $f(x) = \frac{2x}{x+1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Answer (4): $f'(x) =$__________
(5). Find the following derivatives (You do not need to simplify your answers).

(a) (4 points) \( f(x) = 13^{3x^2-8x+5} \)

Answer (5a): \( f'(x) = \) 

(b) (4 points) \( g(x) = \frac{e^{3x} - 7x^8}{x^3} \)

Answer (5b): \( g'(x) = \) 

(c) (4 points) \( f(x) = (5x^6 + 3)^5(3 - \ln x) \)

Answer (5c): \( f'(x) = \)
(Problem (5) cont.)

(d) (5 points) \( f(x) = \sqrt[3]{(x + 2)^{15}(x - 5)^6} \)

Answer (5d): \( f'(x) = \) ________________________

(e) (5 points) \( y = \log_{13} [(4x - 5)^6] \)

Answer (5e): \( y' = \) ________________________
(6). Let \( f(x) = 2x^3 - 3x^2 - 72x + 16 \).

(a) (10 points) Use the Second Derivative Test to find where the relative maximum(s) and the relative minimum(s) of \( f(x) \) occur.

Answer (6a): rel. max(s) at \( x = \) _________________
rel. min(s) at \( x = \) _________________

(b) (5 points) Find where the absolute maximum and absolute minimum for \( f(x) \) occur over the interval \([-7, 3]\).

Answer (6b): absolute max(s) at \( x = \) _________________
absolute min(s) at \( x = \) _________________
Let \( f(x) = 3x^4 - 3x^3 + 3 \).

(a) (15 points) Use derivatives and a sign graph to determine the interval(s) on which \( f(x) \) is increasing and on which \( f(x) \) is decreasing AND indicate where \( f(x) \) has relative maximum and relative minimum points. (If there are none, please say so).

Answer (7a): increasing: ______________________

decreasing: ______________________

rel. max. points(s) at \( x = \) ______________________

rel. min. point(s) at \( x = \) ______________________

(b) (15 points) Use derivatives and a sign graph to determine the interval(s) on which \( f(x) \) is concave up and on which \( f(x) \) is concave down AND indicate where \( f(x) \) has inflection point(s). (If there are none, please say so).

Answer (7b): concave up: ______________________

concave down: ______________________

inflection point(s) at \( x = \) ______________________
A company manufactures and sells $q$ headphones per month. The monthly demand function is 

\[ p = 95 - 0.05q \]

and monthly cost function is

\[ c = 700 + 35q \]

(a) (5 points) Find the profit $P(q)$ from producing $q$ headphones in a given month.

Answer (8a): $P(q) =$ 

(b) (12 points) How many headphones should be produced to maximize the monthly profit?

Answer (8b): Headphones: 

(c) (3 points) What is the maximum monthly profit?

Answer (8c): Maximum profit: 

(9). Find the indefinite or definite integral:

(a) (8 points)
\[
\int \frac{1}{5 + 2x^6(12x^5)} \, dx
\]

Answer (9a): ____________________________

(b) (10 points)
\[
\int_{0}^{4} 28x^3e^{x^4} \, dx
\]

Answer (9b): ____________________________
(10). (15 points) A manufacturer’s marginal-cost function is

\[ \frac{dc}{dq} = 0.008q^2 - 0.7q + 80 \]

If \( c \) is in dollars, determine the cost involved to increase production from 90 to 180 units.

Answer (10): Change in cost: _________________
(11). (15 points) Set-up, but DO NOT EVALUATE, an integral to find the area of the region bounded by the given curves. Be sure to find any needed points of intersection.

\[ y = 8 + 4x - x^2 \quad \text{and} \quad y = x^2 - 2x \]

Answer (11): area: ___________________________
The demand equation for a product is

\[ p = 0.01q^2 - 1.1q + 30 \]

and the supply equation is

\[ p = 0.01q^2 + 8 \]

(a) (5 points) Find the equilibrium point \((p_0, q_0)\).

Answer (12a): \(p_0 = \) ________________

\[ q_0 = \) ________________

(b) (15 points) Determine the consumer’s surplus under market equilibrium.

Answer (12b): consumer’s surplus: ________________