Math 1148: Midterm Exam 1

Instructions:

- Show ALL work to receive full credit. Answers with insufficient supporting work will receive little or no credit.

- Please CIRCLE your answers

- If you find the solution to a problem using a graph from your calculator (where appropriate), you need to sketch that graph and label all relevant information.

- The exam consists of 11 problems starting on page 2 and ending on page 6. Make sure your exam is not missing any pages before you start.

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<tr>
<td>Maximum Points</td>
<td>25</td>
<td>15</td>
<td>20</td>
<td>18</td>
<td>22</td>
<td>100</td>
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<tr>
<td>Student Score</td>
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1. Solve the following absolute value inequality: \( |2x - 3| - 7 < 5 \) (8 points)

2. Determine the domains of the following functions. (3 points each)

<table>
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<th>Functions</th>
<th>Domains (use interval notation)</th>
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<tr>
<td>( f(x) = \frac{x^2 - 7}{x - 3} )</td>
<td></td>
</tr>
<tr>
<td>( g(x) = \frac{x - 7}{x^2 - 9} )</td>
<td></td>
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<tr>
<td>( h(x) = \frac{\sqrt{x + 3}}{x - 7} )</td>
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3. Write an equation for a line which passes through the point (3,5) and is perpendicular to the line \( 4x + 7y = 17 \). (8 points)
4. The temperature of a hot cup of coffee as it cools on a counter can be modelled by the following function where ‘t’ is minutes that it has been cooling and ‘T’ is temperature in degrees Fahrenheit.

\[ T(t) = 72 + 120(1.065)^{-t} \]

Use this function to find the average rate at which the coffee is cooling from \( t = 3 \) minutes to \( t = 10 \) minutes. (5 points)

5. A small business makes cupcakes and sells them at the local market each week. There is a fixed weekly cost to sell at the market of $325. The total of all costs involved in making the cupcakes amounts to $0.43 per cupcake. You plan to sell each box of 8 cupcakes for $5.00.

a. Write a linear revenue function \( P(x) \) for the total profit of your business if you sell \( x \) boxes of cupcakes in a given week. (6 points)

b. How many boxes do you need to sell in order to earn a profit of $500. (Round up to the nearest box)? (4 points)
6. Suppose that a chemist needs to have 25 liters of a salt water solution that is 14% salt. She needs to combine some 8% salt solution with some 20% salt solution. How many liters of each solution should she mix together? (10 points)

7. Suppose you sell tickets for the county fair, children cost $3.00 each, adults cost $7.00 each and seniors cost $5.00. There were half as many seniors as adults and there were 1200 total tickets sold. How many adult tickets did you sell if the total revenue from tickets was $6800?
   a. Set up a system of three equations to model this problem. (4 points)
   
   b. Solve the system showing your work. (6 points)
8. Starting with the basic function \( f(x) = \sqrt{x} \), the function \( g(x) \) is obtained by applying the following transformations to \( f(x) \).

First, shift the graph 5 units to the left. Then reflect the graph over the x-axis.
Stretch the graph vertically by a factor of 2. Finally, shift the graph up by 3 units.

a. Find the formula for \( g(x) \)  
   (4 points)

b. Sketch the graph of \( y = g(x) \). Plot and label at least two points on the transformed graph. 
   (6 points)

9. For each equation below, determine whether or not \( y \) is a function of \( x \). Circle the ones that are.  
   (2 points each)

a. \( x^3 + 3y^2 = 7 \)
b. \( x^2 + 3y^3 = 7 \)
c. \( x + 3|y| = 7 \)
d. \( |x + 3y| = 7 \)
10. For the function below, answer the following:
   (3 points each)
   Use interval notation to write the intervals over which \( f(x) \) is
   Increasing –
   Decreasing –
   Also identify the location and value of any
   Relative Maxima –
   Relative Minima –

11. The piece-wise defined function \( P(x) \) is given by:
\[
P(x) = \begin{cases} 
  x + 5 & \text{if } x \leq -3 \\
  x^2 - 9 & \text{if } -3 < x \leq 1 \\
  4 & \text{if } x > 1 
\end{cases}
\]
   a) Find the following: (4 points)
   \[ P(-6) = \quad P(-2) = \quad P(1) = \quad P(7) = \]
   b) Plot and label your points from part a) and then sketch the graph of \( y = P(x) \). (6 points)