Math 1148: Final Exam

Instructions:

- Show ALL work to receive full credit. Answers with insufficient supporting work will receive little or no credit.

- Please CIRCLE your answers

- If you find the solution to a problem using a graph from your calculator *(where allowed)*, you need to sketch that graph and label all relevant information.

- The exam consists of 16 problems starting on page 2 and ending on page 10. Make sure your exam is not missing any pages before you start.

Some Formulas that may be Useful:

\[
A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = Pe^{rt} \quad A = P(a)^t \quad A = P(1 + r)^t
\]

\[
M = \log \left(\frac{i}{s}\right) \quad T(t) = T_s + De^{-kt} \quad m(t) = m_02^{\left(-\frac{t}{h}\right)}
\]

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1. Solve the following inequality. *Give your answers in interval notation.*

\[
\frac{3x}{x-5} \leq x
\]  
(12 points)

2. Divide the following polynomials and label the quotient and remainder. (Synthetic division will not work)

\[
\frac{4x^3 - 3x^2 + x - 7}{x^2 - 5}
\]  
(10 points)

3. Consider the quadratic function \( f(x) = 3x^2 - 18x + 21 \). Complete the square to express the function in standard form.  

(10 points)
4. Find the inverse functions of the following two functions:
   
   a. Given \( f(x) = \sqrt[3]{5x} - 2 \). Find the inverse function, \( f^{-1}(x) \)  
      (8 points)

   b. Given \( g(x) = \frac{7x + 1}{3 - 5x} \). Find the inverse function, \( g^{-1}(x) \)  
      (8 points)

5. A certain car gets approximately 26 miles/gallon on the highway and 19 miles/gallon in the city. Suppose you drove a total of 349.5 miles on a full tank (16 gallons). Let \( x = \) # of gallons used while driving on highway and \( y = \) # of gallons used for city driving. Set up and solve the system of equations. Show the algebra needed for full credit.  
      (10 points)

   Gallons used on Highway = ______  
   Gallons used in City = __________

   Miles driven on Highway = ________  
   Miles driven in City = __________
6. Given the function \( f(x) = 3x - \sqrt{x} \) find the average rate of change of \( f(x) \) from \( x = 8 \) to \( x = 13 \). Show the ratio and round the final answer to two decimal places. (10 points)

7. Given the function \( f(x) = \frac{5}{x} \), find the average rate of change from \( x = a \) to \( x = a + h \).
   Simplify as much as possible and show your work. (10 points)

8. Suppose a certain company determines that if it sets the price of an item at ‘\( p \)’ dollars, then it can sell a quantity given by \( q = 400 - 8p \) of the item.
   a. Based on this find a function for the revenue from sales of this item in terms of \( p \). (8 points)

   b. Find the price ‘\( p \)’ that will maximize the revenue for the company.
      Draw a rough sketch of the graph of revenue with respect to price. (6 points)
9. Consider the rational function shown in the graph.

\[ r(x) = \]

a. Find the $x$-intercept(s), if any \hspace{1cm} (6 points)

b. Find the equations of any vertical asymptotes \hspace{1cm} (4 points)

c. Find the equation of the horizontal asymptote \hspace{1cm} (you can assume it has one) \hspace{1cm} (4 points)

d. Write a rational function for the graph: \hspace{1cm} (8 points)
10. Solve the following system of three equations in three variables. 
   (12 points) 
   Show your work.

   \[ x + 2y - z = 11 \]
   \[ 3x - y + 2z = 8 \]
   \[ 3x + 3z = 15 \]
11. Suppose $3000 is deposited into an account that offers 6% annual interest compounded quarterly. How long until the account is worth $10,000. Round your answer to the nearest tenth of a year. (12 points)

12. Assuming \( x \) is in the domain, expand the expression below as much as possible. (12 points)

\[
\log \left( \frac{(x - 5)\sqrt{x + 2}}{(x - 4)(x + 3)^2} \right)
\]
13. The population of a certain town is currently 27,000 people and is doubling every 4 years. When will the population reach 90,000 people? *Round your answer to the nearest tenth of a year.* (12 points)

14. Algebraically solve for $x$. Show all work.
\[ \log_2(x - 3) + \log_2(x - 1) = 3 \] (12 points)
15. Suppose one City with a population of 20,000 has an annual growth rate of 9%, while another city has a population of 55,000 and an annual growth rate of only 6%. Solve the following equation to determine when they will have the same population.

\[20000(1.09)^x = 55000(1.06)^x\]  

(14 points)

*Your final answer must involve logarithms. Decimal approximations will receive at most 8 points no matter how much work is shown.*
16. The Richter scale is \( M = \log \frac{I}{S} \) where \( M \) is the magnitude of the earthquake, \( I \) is the intensity of the earthquake at the epicenter, and \( S \) is the intensity of the “standard” earthquake.

a. Change the equation \( M = \log \frac{I}{S} \) from logarithmic form to exponential form and then solve for “I”. (6 points)

b. A recent earthquake in Alaska had a magnitude of 7.1 on the Richter scale. At the OSU-Michigan football game a few weeks ago, the OSU seismologists measured the vibrations of the stadium after the winning play at a magnitude of 5.79. How many times more intense was the Alaska earthquake than the Ohio ‘earthquake’? (In other words, what is the ratio of their intensities?) (6 points)