INSTRUCTIONS

• SHOW ALL WORK! Incorrect answers with work shown may receive partial credit, but unsubstantiated answers may receive NO credit.

• Give EXACT answers unless asked to do otherwise.

• You do not need to simplify numerical answers such as \( \frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}} \).

• Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA’s, laptops, and cell phones are prohibited. Do not have these devices out!

• The exam duration is 55 minutes.

• The exam consists of 5 problems starting on page 2 and ending on page 8. Make sure your exam is not missing any pages before you start.

<table>
<thead>
<tr>
<th>PROBLEM NUMBER</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(32)</td>
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<td>2</td>
<td>(24)</td>
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<td>3</td>
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<td>4</td>
<td>(24)</td>
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<td>5</td>
<td>(10)</td>
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<td>TOTAL</td>
<td>(100)</td>
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1. (32 pts)

The graph of a function $f$ is given in the figure below. Use the graph of $f$ to answer the questions below.

(I) (4 pts) Find the domain of $f$.

Domain of $f =$

(II) (4 pts) Find the range of $f$.

Range of $f =$

(III) Find the following values.

(Note: Possible answers include $+\infty$ or $-\infty$.)

(a) (2 pts) $\lim_{x \to 3^+} f(x) =$

(b) (2 pts) $\lim_{x \to 0} f(x) =$
1. (CONTINUED)

(c) (2 pts) \( f(0) = \)

(d) (2 pts) \( f(4) = \)

(e) (2 pts) \( \lim_{x \to 4^-} f(x) = \)

(f) (2 pts) \( \lim_{x \to 4^+} f(x) = \)

(g) (2 pts) \( \lim_{x \to 4} f(x) = \)

(IV) (2 pts) Is \( f \) one-to-one? Why?

(V) (4 pts) Is \( f \) continuous at \( a = 0 \)?
Use the definition of continuity to explain your answer.

(VI) (4 pts) Determine the intervals of continuity for \( f \).
2. (24 pts)

Evaluate the limit. Show your work.
(Note: Possible answers include $+\infty$ or $-\infty$.)

You may NOT use a table of values, a graph, or L'Hôpital's Rule to justify your answer.

(a) $\lim_{x \to 2} \frac{\sqrt{3x - 2} - 2}{x - 2} =$

(b) $\lim_{x \to -\infty} \frac{x^5 - 2x^3 + 2}{x^4 - 3} =$
(c) \( \lim_{x \to 0^+} x^3 \cos \left( \frac{\pi}{x} \right) = \)
3. (10 pts)

(I) Suppose $s(t)$ is the position of an object moving along a line at time $t \geq 0$.
What is the average velocity between the times $t = a$ and $t = b$?

$$v_{av} =$$

(II) The table gives the position $s(t)$ of an object moving along a line at time $t$, over a three-second interval.
Find the average velocity of the object over the interval $[1, 3]$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>1.2</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(t)$</td>
<td>0</td>
<td>22</td>
<td>32</td>
<td>48</td>
<td>60</td>
<td>68</td>
<td>72</td>
</tr>
</tbody>
</table>
Let \( g(x) = \begin{cases} \frac{x}{x + 2} & \text{if } x < 0 \text{ and } x \neq -2 \\ e^{-x} + c & \text{if } x \geq 0. \end{cases} \)

(I) Evaluate the limit. Show your work.
(Note: possible answers may include \(-\infty\) or \(\infty\).)

(a) (4 pts) \( \lim_{x \to -2^-} g(x) = \)  
(b) (4 pts) \( \lim_{x \to -2^+} g(x) = \)

(c) (2 pts) \( \lim_{x \to -\infty} g(x) = \)  
(d) (2 pts) \( \lim_{x \to +\infty} g(x) = \)

(II) (2 pts) Find (the equations of) all vertical asymptotes.

(III) (4 pts) Find (the equations of) all horizontal asymptotes.

(IV) (6 pts) Determine the value of the constant \( c \) for which

\[ \lim_{x \to 0} g(x) \text{ exists.} \]
5. (10 pts)

Explain how the Intermediate Value Theorem can be used to show that the equation

\[ x^3 - 4x + 2 = 0 \]

has a solution on the interval \([0, 1]\).