# TOPOLOGICAL SPACES ASSOCIATED TO A MODULE

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A topological space is it said to be *spectral* if it satisfies that is $T_0$, quasicompact, has a basis of compact open subsets which is closed under finite intersection, and all irreducible closed subsets are closures of points (i.e. sober). In *Prime ideal structure in commutative rings,Trans. Amer. Math. Soc. 142 (1969)* M. Hochster characterized spectral topological spaces showing that a topological space $X$ is spectral if and only if it is homeomorphic to $Spec(R)$ for some commutative ring $R$. Inspired by that result, we are interesting in the behavior of a spectrum for a module $M$.

In [MSZ15], we studied a prime spectrum for a module throught some associated frames, and we gave a module counterpart of the well-known result that in a commutative ring the set of semiprime ideals, that is, radical ideals is a frame. In [MMSZ17], we continue this work, we define semiprimitive submodules and we prove that they form a spatial frame canonically isomorphic to the topology of Max(M). Also, we study the soberness of a prime spectrum for $M$ and for the subspace Max(M) in terms of the point space of that frame.

The purpose of this talk is to present some of these results. This is a jointly work with M. Medina-Barcenas, L. Morales-Callejas and A. Zaldivar-Corichi.

## REFERENCES

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