6000-Level Analysis Qualifying Exams Syllabus, Summer 2018

These exams are intended to test students' proficiency with the core material that is covered in Math 6211 (Real Analysis I) and Math 6212 (Real Analysis II). The core material is the material that all instructors for this sequence should cover. Different instructors may cover different additional topics, as time permits and according to their taste, to illustrate the uses of the core material. For the purpose of this exam syllabus, the main reference is Gerald B. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed., Wiley, 1999. Prerequisite material includes basic analysis at the level of Walter Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, 1976, as covered in Math 5201 (Introduction to Real Analysis I) and Math 5202 (Introduction to Real Analysis II), plus basics of set theory and metric spaces, as reviewed in Chapter 0 of Folland. The following syllabus specifies the sections of Folland that treat the core material for 6211 and 6212 and that students should be familiar with for the 6000-level analysis qualifying exams. This syllabus applies for Summer 2018. The division of core topics between 6211 and 6212 varies from year to year and this syllabus may be revised slightly each year.

Math 6211, Autumn 2017.

1 Measures
- 1.1 Introduction
- 1.2 σ-algebras
- 1.3 Measures
- 1.4 Outer Measures
- 1.5 Borel Measures on the Real Line

2 Integration
- 2.1 Measurable Functions
- 2.2 Integration of Nonnegative Functions
- 2.3 Integration of Complex Functions
- 2.4 Modes of Convergence
- 2.5 Product Measures
- 2.6 The n-dimensional Lebesgue Integral
- 2.7 Integration in Polar Coordinates

3 Signed Measures and Differentiation
- 3.1 Signed Measures
- 3.2 The Lebesgue-Radon-Nikodym Theorem
- 3.3 Complex Measures
- 3.4 Differentiation [of Measures] on Euclidean Space
- 3.5 Functions of Bounded Variation

4 Point Set Topology
- 4.1 Topological Spaces
- 4.2 Continuous Maps

5 Elements of Functional Analysis
- 5.1 Normed Vector Spaces
- 5.2 Linear Functionals

6 L^p Spaces
- 6.1 Basic Theory of L^p Spaces
- 6.2 The Dual of L^p
- 6.3 Some Useful Inequalities

Math 6212, Spring 2018.

4 Point Set Topology
  4.3 Nets
  4.4 Compact Spaces
  4.5 Locally Compact Hausdorff Spaces
  4.6 Two Compactness Theorems

5 Elements of Functional Analysis
  5.3 The Baire Category Theorem and its Consequences
  5.5 Hilbert Spaces

7 Radon Measures.
  7.1 Positive Linear Functionals on $C_c(X)$
  7.2 Regularity and Approximation Theorems
  7.3 The Dual of $C_0(X)$

8 Elements of Fourier Analysis
  8.1 Preliminaries
  8.2 Convolutions
  8.3 The Fourier Transform
  8.4 Summation of Fourier Integrals and Series
  8.5 Pointwise Convergence of Fourier Series

9 Elements of Distribution Theory
  9.1 Distributions
  9.2 Compactly Supported, Tempered, and Periodic Distributions
  9.3 Sobolev Spaces