Ph.D. Algebra Qualifying Exam
6112

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Directions

1. Initial the roster sheet and enter a code name for yourself that is different from any code name that has already been entered.
2. Answer each question on a separate sheet or sheets of paper, and write your *code name* and the *problem number* on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.
3. Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment you use in this respect is an important part of the exam.
4. This is a two hour, closed book, closed notes exam.

1. Let R be a ring with identity and $N \in Ob(R - mod)$. Show that the contravariant Hom functor $Hom_R(-, N)$ from $R - mod$ to $\mathbb{Z} - mod$ is left exact but not right exact.

2. Let R be a commutative ring (with identity) and let M be an R-module. Let $\mathfrak{a}$ be an ideal of R. Show that $(R/\mathfrak{a}) \otimes_R M \simeq M/\mathfrak{a}M$.

3. Consider $M = \mathbb{Z}/2\mathbb{Z}$ as a module over the ring $R = \mathbb{Z}/4\mathbb{Z}$. Compute $Ext^n_R(M, M)$ for all n.

4. Consider the extension $\mathbb{Q}(\alpha)$ over $\mathbb{Q}$ where $\alpha = \sqrt{2 + \sqrt{2}}$. Show that $\mathbb{Q}(\alpha)$ is Galois over $\mathbb{Q}$ and determine its Galois group.

5. Let F be a field which contains n distinct n^{th} roots of 1. Let E be the splitting field over F of a polynomial

   $$f(x) = (x^n - a_1) \cdots (x^n - a_r) \quad \text{with} \quad a_i \in F.$$

   Show that E/F is an abelian Galois extension such that $G = Gal(E/F)$ has exponent m dividing n.