Syllabus for Core Algebra Sequence Math 6111–2

References: *Algebra* (3rd ed.) by Serge Lang
Basic Algebra (2nd ed.) by Nathan Jacobson (in two volumes, I and II)
Linear Representations of Finite groups by Jean-Pierre Serre
Introduction to Commutative Algebra, by Michael Atiyah and I. G. Macdonald

6111:

Group Theory:

monoids, subgroups, cyclic groups, direct products
homomorphisms, normal subgroups, factor groups
composition series and the Jordan-Hölder Theorem
solvable groups, derived series
semi-direct products, automorphisms
groups acting on sets; permutation groups and matrix groups; simplicity of A_n

Lang: I.1–5
Jacobson I: 1.1–10, 1.12; 4.6
Jacobson II: 3.3

Representations of finite groups:

complex linear representations of finite groups
character theory, orthogonality relations
permutation representations, induced representations, the regular representation

Lang: XVIII.1–6
Serre: I.1–3
Jacobson II: 5.1–4, 5.6–11

Ring Theory:

basic ring theory, commutative rings, polynomial rings;
modules; noetherian rings and modules; Hilbert Basis Theorem; localization;
integral domains, UFDs, PIDs, polynomial rings over UFDs, irreducibility criteria
symmetric polynomials; discriminant

Lang: II.1–3, II.5; IV.1–4, IV.6
Jacobson I: 2.1–7, 2.10–11, 2.13–16
Jacobson II: 7.1–4, 7.9
Atiyah-MacDonald: 1,2,3,7

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Category Theory:

categories, dual categories, universal objects
covariant and contravariant functors
representable functors, natural transformations
universal objects, products and coproducts
inverse limits and direct limits
free abelian groups, free groups, generators and relations

Lang I.10–12, III.10
Jacobson II, 1.1–8, 2.5, 2.9

Homological Algebra:

modules, homomorphisms, the *Hom* functor, direct products and direct sums of modules
abelian categories, free modules, projective and injective modules, tensor products
snake lemma, complexes, homology sequence; projective and injective resolutions, derived functors

Lang III.1–4; XX.1–2, XX.4–6
Jacobson II: 6.1–6

Fields

algebraic extensions, algebraic closure
splitting fields, normal extensions; separable and inseparable extensions, simple extensions
finite fields, perfect fields
independence of characters
Galois theory
cyclotomic extensions, Kummer extensions, radical extensions; solvable extensions
the general equation of n^{th} degree, Abel's Theorem
Galois groups of polynomials with integral coefficients
Galois theory for infinite extensions; Krull topology
transcendental extensions;

Lang V.1–6, VI.1–8, VI.14, VIII.1;
Jacobson I, 4.1, 4.4–9, 4.13–14;
Jacobson II, 8.1–2, 8.6–9, 8.12–13