6000-Level Analysis Qualifying Exams Syllabus, Summer 2015

These exams are intended to test students' proficiency with the core material that is covered in Math 6211 (Real Analysis I) and Math 6212 (Real Analysis II). The core material is the material that all instructors for this sequence should cover. Different instructors may cover different additional topics, as time permits and according to their taste, to illustrate the uses of the core material. In 2014/2015, all of 6211 and all but the last four weeks of 6212 were devoted to covering core material. For the purpose of this exam syllabus, the main reference is Gerald B. Folland, *Real Analysis: Modern Techniques and Their Applications*, 2nd ed., Wiley, 1999. Prerequisite material includes basic analysis at the level of Walter Rudin, *Principles of Mathematical Analysis*, 3rd ed., McGraw-Hill, 1976, as covered in Math 5201 (Introduction to Real Analysis I) and Math 5202 (Introduction to Real Analysis II), plus basics of set theory, metric spaces, and point-set topology as reviewed in Chapters 0 and 4 of Folland. The following syllabus specifies the sections of Folland that treat the core material for 6211 and 6212 and that students should be familiar with for the 6000-level analysis qualifying exams. This syllabus applies for Summer 2015. The division of core topics between 6211 and 6212 may vary from year to year and accordingly, this syllabus may be revised slightly each year.

Math 6211, Autumn 2014.

1 Measures
- 1.1 Introduction
- 1.2 σ-algebras
- 1.3 Measures
- 1.4 Outer Measures
- 1.5 Borel Measures on the Real Line

2 Integration
- 2.1 Measurable Functions
- 2.2 Integration of Nonnegative Functions
- 2.3 Integration of Complex Functions
- 2.4 Modes of Convergence
- 2.5 Product Measures
- 2.6 The n-dimensional Lebesgue Integral
- 2.7 Integration in Polar Coordinates

5 Elements of Functional Analysis
- 5.1 Normed Vector Spaces
- 5.3 The Baire Category Theorem and its Consequences [covered in 6211: The Uniform Boundedness Principle] [deferred to 6212: Open Mapping Theorem, Closed Graph Theorem]
- 5.5 Hilbert Spaces [5.19, 5.20, 5.21, 5.23, 5.26, 5.27, 5.29, 5.30] [other items deferred to 6212]

6 L^p Spaces
- 6.1 Basic Theory of L^p Spaces [Minkowski's Inequality, Hölder's Inequality, Inclusions Between L^{p_1} and L^{p_2}]
- 6.3 Some Useful Inequalities

8 Elements of Fourier Analysis
- 8.1 Preliminaries
- 8.2 Convolutions
- 8.3 The Fourier Transform
- 8.4 Summation of Fourier Integrals and Series
- 8.5 Pointwise Convergence of Fourier Series [also the use of the uniform boundedness principle to show that in the sense of category, most 1-periodic continuous functions on $\mathbf{R}$ have Fourier series which diverge at most points]

6000-Level Analysis Qualifying Exams Syllabus, Summer 2015

Math 6212, Spring 2015.

3 Signed Measures and Differentiation
- 3.1 Signed Measures
- 3.2 The Lebesgue-Radon-Nikodym Theorem
- 3.3 Complex Measures
- 3.4 Differentiation [of Measures] on Euclidean Space
- 3.5 Functions of Bounded Variation [and Associated Measures]

5 Elements of Functional Analysis
- 5.2 Linear Functionals [and the Hahn-Banach Theorem]
- 5.3 The Baire Category Theorem and its Consequences [Open Mapping Theorem, Closed Graph Theorem] [The Uniform Boundedness Principle was already covered in 6211]
- 5.4 Topological Vector Spaces [Weak Topology, Weak* Topology, Alaoglu's Theorem, Strong Operator Topology, Weak Operator Topology]
- 5.5 Hilbert Spaces [5.22, 5.24, 5.25 (the dual of a Hilbert space)] [The other items were already covered in 6211]

6 L^p Spaces
- 6.2 The Dual of L^p

7 Radon Measures.
- 7.1 Positive Linear Functionals on $C_c(X)$
- 7.2 Regularity and Approximation Theorems
- 7.3 The Dual of $C_0(X)$

8 Elements of Fourier Analysis
- 8.6 Fourier Analysis of Measures

9 Elements of Distribution Theory
- 9.2 Periodic Distributions and their Fourier Series [The other parts of 9.2 were not covered]

In Spring 2015, in the last four weeks of 6212, the following topics were discussed: Banach algebras, C^* algebras, and the spectral theorem for a bounded normal operator on a Hilbert space. These are not core topics of the sequence and they will not be tested on the qualifying exam for 6212 in August 2015.