# Algebra Qualifying Exam II
# 2019

## Directions

Put your name and the last four digits of your Social Security Number on the roster sheet when you receive it and enter a code name for yourself that is different from any code name that has already been entered.

Answer each question on a separate sheet or sheets of paper, and write your code name and the problem number on each sheet of paper that you submit for grading. Do not put your real name on any sheet of paper that you submit for grading.

Answer as many questions as you can. Do not use theorems which make the solution to the problem trivial. Always clearly display your reasoning. The judgment you use in this respect is an important part of the exam.

This is a closed book, closed notes exam.

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1. Prove that the inverse limit of monomorphisms is monic. Prove that the direct limit of epimorphisms is epi.

2. Let $R$ be a commutative ring with 1 and let $S \subset R$ be a multiplicatively closed set. Prove that $M \mapsto S^{-1}M$ is an exact functor from ($R$-mod) to ($S^{-1}R$-mod).

3. In the category of abelian groups, show that $\mathrm{Ext}^n = 0$ for $n > 1$.

4. Let $E/F$ be a finite extension of finite fields. Show that the norm map $N_{E/F} : E^\times \to F^\times$ is surjective.

5. Prove the primitive element theorem for finite Galois extensions: if $E/F$ is finite Galois, then $E = F(\alpha)$ for some $\alpha \in E$.