New triangles from old

"What is . . . ?" Seminar. D. Shapiro

Wednesday, 3:30 PM, June 22, 2011.

Triangle $\mathcal{T} = \Delta ABC$ has midpoint-triangle $\mathcal{M}_{1/2}(\mathcal{T}) = A'B'C'$. Here A' is the midpoint of side AB, B' is the midpoint of BC, etc. More generally, let $\mathcal{M}_r(\mathcal{T})$ be the triangle built from the r-division-points of the sides of $\mathcal{T}$.

- Does $\mathcal{M}_r^k(\mathcal{T})$ converge to a point as $k \to \infty$?

It's easy to see that $\mathcal{M}_{1/2}(\mathcal{T})$ is a triangle with sides parallel those of $\mathcal{T}$. It is somewhat surprising that a similar result holds when the one-third-point operation is repeated:

Sides of $\mathcal{M}_{1/3}\big(\mathcal{M}_{1/3}(\mathcal{T})\big)$ are parallel to the sides of $\mathcal{T}$. (Exercise: Find an elementary proof.)

- Does this generalize to repetitions of $\mathcal{M}_{1/4}$, or $\mathcal{M}_{1/5}$?

- Do similar results hold true for more general polygons?

In this lecture I will describe how to translate those questions into linear algebra .