GEOMETRY AND THE QUANTUM

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Abstract. In collaboration with A. Chamseddine and S. Mukhanov, motivated by the construction of spectral manifolds in noncommutative geometry, we introduce a higher degree Heisenberg commutation relation involving the Dirac operator and the Feynman slash of scalar fields. This commutation relation appears in two versions, one sided and two sided. It implies the quantization of the volume. In the one-sided case it implies that the manifold decomposes into a disconnected sum of spheres which will represent quanta of geometry. The two sided version in dimension 4 predicts the two algebras $M_2(\mathbb{H})$ and $M_4(\mathbb{C})$ which are the algebraic constituents of the Standard Model of particle physics. This taken together with the non-commutative algebra of functions allows one to reconstruct, using the spectral action, the Lagrangian of gravity coupled with the Standard Model. We show that any connected Riemannian Spin 4-manifold with quantized volume > 4 (in suitable units) appears as an irreducible representation of the two-sided commutation relations in dimension 4 and that these representations give a seductive model of the "particle picture" for a theory of quantum gravity in which both the Einstein geometric standpoint and the Standard Model emerge from Quantum Mechanics.