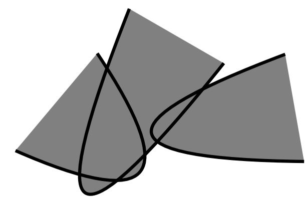
# 2015 Gordon examination problems

**1.** Prove that there are infinitely many integers not representable as $n^3+m^3+k^3$, where $n$, $m$, $k$ are positive integers.

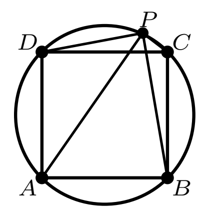
**2.** Can the plane be covered by the interiors of a finite collection of parabolas? (The parabolas may have any orientation, see picture.)



**3.** Let $A$ be a $2\times 2$ matrix with integer entries and determinant 1. Prove that $A$ is a product of several matrices of the form $\left(\begin{smallmatrix}1&1\\0&1\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&0\\1&1\end{smallmatrix}\right)$, $\left(\begin{smallmatrix}1&-1\\0&1\end{smallmatrix}\right)$, and $\left(\begin{smallmatrix}1&0\\-1&1\end{smallmatrix}\right)$.

**4.** Let $p\in\mathbb{Z}[x]$ (a polynomial with integer coefficients) and let $a,b,c\in\mathbb{Z}$ be such that $p(a)=b$, $p(b)=c$, and $p(c)=a$. Prove that $a=b=c$.

**5.** The square $ABCD$ is inscribed in a circle of radius $R$, and $P$ is a point on the circle. Prove that $|PA|^4+|PB|^4+|PC|^4+|PD|^4=24R^4$.




**6.** Prove that for any $x,y,z\geq 0$, $\sqrt[2]{x+\sqrt[3]{y+\sqrt[4]{z}}}\geq \sqrt[32]{xyz}$.