Worksheet #10 Solutions

1. a) $2\vec{u} - 3\vec{v} = 2[3\hat{i} - \hat{j} + k] - 3[-\hat{i} + 2\hat{k}]$

$= 6\hat{i} - 2\hat{j} + 2\hat{k} + 3\hat{i} - 6\hat{k}$

$= \boxed{9\hat{i} - 2\hat{j} - 4\hat{k}}$

b) $|\vec{u}| = \sqrt{3^2 + (-1)^2 + (1)^2} = \boxed{\sqrt{11}}$

c) $|\vec{v} + 2\vec{w}|$

- First, note: $\vec{v} + 2\vec{w} = -\hat{i} + 2\hat{k} + 2(6\hat{i} + \hat{j} - \hat{k}) = 11\hat{i} + 2\hat{j}$
- Thus, $|\vec{v} + 2\vec{w}| = \sqrt{11^2 + 2^2} = \boxed{\sqrt{125}}$ or $\boxed{5\sqrt{5}}$

d) To find a unit vector, calculate the magnitude, then divide the original vector by that magnitude:

- $|\vec{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$
- $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{1}{\sqrt{5}}[-\hat{i} + 2\hat{k}] = \boxed{-\dfrac{1}{\sqrt{5}}\hat{i} + \dfrac{2}{\sqrt{5}}\hat{k}}$

e)
- $\vec{u} + 2\vec{w} = 3\hat{i} - \hat{j} + \hat{k} + 2[6\hat{i} + \hat{j} - \hat{k}] = 15\hat{i} + \hat{j} - \hat{k}$
- $|\vec{u} + 2\vec{w}| = \sqrt{15^2 + 1^2 + (-1)^2} = \sqrt{227}$
- We want a vector $\vec{a}$ parallel to $\vec{u} + 2\vec{w}$, so $\vec{a} = c(\vec{u} + 2\vec{w})$.

Since $|\vec{a}| = 2 = |c||\vec{u} + 2\vec{w}|$

$2 = |c|\sqrt{227}$

$|c| = \dfrac{2}{\sqrt{227}}$.

Since $\vec{a} \parallel \vec{u} + 2\vec{w}$, $c > 0$, so $c = \dfrac{2}{\sqrt{227}}$ and $\boxed{\vec{a} = \dfrac{2}{\sqrt{227}}[15\hat{i} + \hat{j} - \hat{k}]}$

f)
- $\vec{w} = \sqrt{6^2 + 1^2 + (-1)^2} = \sqrt{38}$.
- Since $\vec{a}$ is parallel to $\vec{v}$, $\vec{a} = c\vec{v}$, and since $|\vec{a}| = |\vec{w}|$;

$|\vec{w}| = |c||\vec{v}|$

We found $|\vec{v}| = \sqrt{5}$ in d), so:

$$\sqrt{38} = c\sqrt{5}$$

$$c = \sqrt{\frac{38}{5}}$$

Hence, $\vec{a} = c\vec{v} = \boxed{\sqrt{\frac{38}{5}}\,[-\hat{\imath} + 2\hat{k}]}$

2. • Note: $|\vec{v}| = \sqrt{(-1)^2+1^2} = \sqrt{2}$, $|\vec{w}| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

• Since all vectors $\vec{u}$ must be parallel to $\vec{w}$, $\vec{u} = c\vec{w}$

Thus: $|\vec{u}| = |c|\,|\vec{w}|$, and since $|\vec{u}| = |\vec{v}|$:

$$\sqrt{2} = |c|\sqrt{2}$$

$$|c| = 1$$

Since $\vec{u} \parallel \vec{w}$, $c > 0$, so $\underline{c = 1}$. Hence $\vec{u} = c\vec{w} \Rightarrow \boxed{\vec{u} = \vec{w}}$

There is only 1 vector!

3 a) <u>True</u>; equal vectors have equal magnitudes and are in the same direction

b) <u>False</u>; vectors must have equal magnitudes <u>AND</u> directions!

c) <u>False</u>; $|c\vec{u}| = \sqrt{(cu_1)^2 + (cu_2)^2 + (cu_3)^2} = \sqrt{c^2(u_1^2+u_2^2+u_3^2)}$

$$= |c|\sqrt{u_1^2+u_2^2+u_3^2}$$

$$= |c|\,|\vec{u}|.$$

d) <u>True</u>; two vectors are equal iff their components are equal.

4. a) $\vec{u}\cdot\vec{v} = (1)(-1) + (-1)(2) + (2)(-1) = -1-2-2 = \boxed{-5}$

b) $2\vec{u}\cdot 3\vec{v} = 6(\vec{u}\cdot\vec{v}) = 6(-5) = \boxed{-30}$

c) $\vec{u}+\vec{v} = \hat{\jmath} + \hat{k}$, so $\vec{u}\cdot(\vec{u}+\vec{v}) = (1)(0) + (-1)(1) + (2)(1). = \boxed{1}$

d) $|\vec{u}| = \sqrt{1^2+(-1)^2+2^2} = \sqrt{6}$, $|\vec{v}| = \sqrt{(-1)^2+2^2+(-1)^2} = \sqrt{6}$.

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$-5 = \sqrt{6}\sqrt{6}\cos\theta$$

$$\cos\theta = -\frac{5}{6} \rightarrow \boxed{\theta = \cos^{-1}\left(-\frac{5}{6}\right)}$$

f) $\text{proj}_{\vec{v}} \vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}} \vec{v}$. We have $\vec{u}\cdot\vec{v} = -5$ from a), and:

$$\vec{v}\cdot\vec{v} = (-1)^2 + 2^2 + (-1)^2 = 6, \text{ so:}$$

$$\text{proj}_{\vec{v}} \vec{u} = -\frac{5}{6}\left[-\hat{i} + 2\hat{j} - \hat{k}\right]$$

$$\vec{u}\cdot \text{proj}_{\vec{v}} \vec{u} = -\frac{5}{6}\left[1(-1) + (-1)(2) + 2(-1)\right] = \boxed{\frac{25}{6}}$$

5. $\text{proj}_{\vec{u}} \vec{u} = \dfrac{\vec{u}\cdot\vec{u}}{\vec{u}\cdot\vec{u}} \vec{u} = \vec{u}$!

This should make sense, since the part of any vector $\vec{u}$ that lies along itself is precisely $\vec{u}$!

6a) $\vec{u} \perp \vec{v}$ iff $\vec{u}\cdot\vec{v} = 0$. Note: $\vec{u}\cdot\vec{v} = (-1)(2) + (3)(1) = 1$

Hence, $\vec{u}$ and $\vec{v}$ are NOT orthogonal!

b) Let $\vec{v} = \langle a, b\rangle$ be a vector orthogonal to $\vec{u}$.

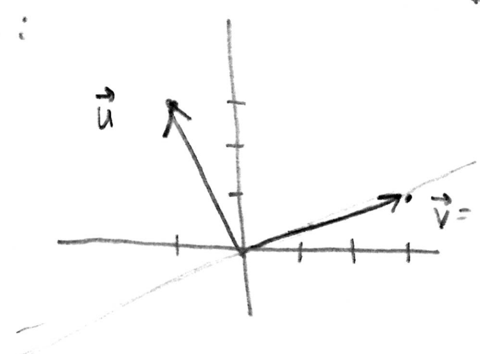
$$\vec{u}\cdot\vec{v} = \langle -1, 3\rangle \cdot \langle a, b\rangle = 0$$

$$-a + 3b = 0$$

$$a = 3b.$$

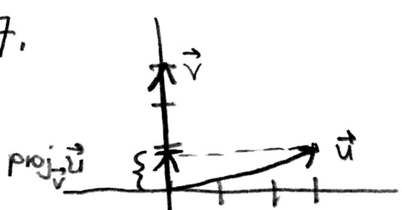
Hence $\vec{v} = \langle a, b\rangle = \langle 3b, b\rangle = \boxed{b\langle 3, 1\rangle, \quad b \in \mathbb{R}}$

c) Pictorally:



All vectors $\perp$ to $\vec{u}$ should lie on the line shown! The b parameter extends / contracts the vector $\vec{v} = \langle 3, 1\rangle$ but preserves its orientation to $\vec{u}$.

7.



Pictorally / Geometrically, $\text{proj}_{\vec{v}} \vec{u} = \hat{j}$

Algebraically: $\text{proj}_{\vec{v}} \vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}} \vec{v} = \dfrac{3}{9}\cdot\left[\hat{i} + 3\hat{j}\right]$

$= \hat{j}$

8. We must show 2 things:

i) If $\vec{v}$ and $\vec{w}$ are parallel, then $\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{\vec{w}}\vec{u}$.

Pf: If $\vec{v}$ and $\vec{w}$ are parallel, there is some $c>0$ s.t.

$$\vec{v} = c\vec{w}.$$

Thus, $\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{c\vec{w}}\vec{u} = \dfrac{\vec{u}\cdot c\vec{w}}{c\vec{w}\cdot c\vec{w}}(c\vec{w})$

$$= \frac{\cancel{c}}{\cancel{c^2}} \frac{\vec{u}\cdot\vec{w}}{\vec{w}\cdot\vec{w}} \cancel{c}\vec{w}$$

$$= \text{proj}_{\vec{w}}\vec{u}.$$

ii) If $\text{proj}_{\vec{v}}\vec{u} = \text{proj}_{\vec{w}}\vec{u}$, then $\vec{v},\vec{w}$ are parallel

Pf: $\text{proj}_{\vec{v}}\vec{u}$ and $\text{proj}_{\vec{w}}\vec{u}$ are equal vectors, so they have the same direction and magnitude.

But $\text{proj}_{\vec{v}}\vec{u} = \Big(\underbrace{\dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}}_{\text{scalar}}\Big)\vec{v}$ is in the direction of $\vec{v}$!

Likewise, $\text{proj}_{\vec{w}}\vec{u}$ is in the direction of $\vec{w}$. Hence, $\vec{v},\vec{w}$ are in the same direction, so they are parallel

9. a) $\vec{u}\times\vec{v} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 1 & -1 \end{vmatrix} = [(-3)(-1)-(1)(1)]\hat{i} - [2(-1)-(1)(1)]\hat{j}$

$$+ [(2)(1)-(1)(-3)]\hat{k}$$

$$= \boxed{2\hat{i} + 3\hat{j} + 5\hat{k}}$$

b) $\vec{u}\times\vec{v}$ is orthogonal to both $\vec{u}$ and $\vec{v}$, so $\boxed{2\hat{i}+3\hat{j}+5\hat{k}}$ is such a vector.

c) Conceptually: $\text{proj}_{\vec{u}}\vec{u}\times\vec{v} = 0$ because $\vec{u}\perp\vec{u}\times\vec{v}$.

Computationally: $\text{proj}_{\vec{u}}\vec{u}\times\vec{v} = \dfrac{\vec{u}\cdot(\vec{u}\times\vec{v})}{\vec{u}\cdot\vec{u}} \leftarrow \vec{u}\perp\vec{u}\times\vec{v} \Leftrightarrow \vec{u}\cdot(\vec{u}\times\vec{v}) = 0.$

Note: $\vec{u}\cdot(\vec{u}\times\vec{v}) = 2(2)+(-3)(3)+(1)(5) = 0.$ so $\boxed{\text{proj}_{\vec{u}}\vec{u}\times\vec{v} = 0}$

d) The area of the parallelogram is $|\vec{u}\times\vec{v}| = \sqrt{2^2+3^2+5^2}$
$= \boxed{\sqrt{38}}$

10 a) $\vec{u}\times\vec{v} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} = -\hat{i} - 3\hat{j} + 6\hat{k}.$

$(\vec{u}\times\vec{v})\times\vec{w} = \det\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 6 \\ 1 & 1 & 1 \end{bmatrix} = \boxed{-9\hat{i} + 7\hat{j} + 2\hat{k}}$

b) $\vec{v}\times\vec{w} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{u}\times(\vec{v}\times\vec{w}) = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 1 & 1 & -2 \end{vmatrix} = \boxed{-2\hat{i} + 6\hat{j} + 4\hat{k}}$

* Note that this shows the cross-product is <u>NOT</u> associative! *

c). $\vec{u}\cdot(\vec{v}\times\vec{w})$.

Note $\vec{v}\times\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$ from b) so:

$\vec{u}\cdot(\vec{v}\times\vec{w}) = 3(1) + (-1)(1) + 0(-2). = \boxed{2}$

d) $\text{proj}_{\vec{w}}\, \vec{u}\times\vec{v} = \dfrac{\vec{w}\cdot(\vec{u}\times\vec{v})}{\vec{w}\cdot\vec{w}.}\,\vec{w}$

- $\vec{w}\cdot(\vec{u}\times\vec{v}) = 1(-1) + 1(-3) + 1(6) = 2$
- $\vec{w}\cdot\vec{w} = 1(1) + 1(1) + 1(1) = 3$

So, $\text{proj}_{\vec{w}}\, \vec{u}\times\vec{v} = \frac{2}{3}\vec{w} = \boxed{\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}}$

e) $\vec{u}\times\vec{v}$ is a vector orthogonal to both $\vec{u}$ and $\vec{v}$. We found this in a): $\boxed{-\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}$

f) · $\text{proj}_{3\vec{v}}(\vec{v}\times\vec{w}) = \text{proj}_{\vec{v}}(\vec{v}\times\vec{w})$ since $3\vec{v}$ is parallel to $\vec{v}\times\vec{w}$

· Since $\vec{v}\times\vec{w} \perp \vec{v}$, $\boxed{\text{proj}_{3\vec{v}}\ \vec{v}\times\vec{w} = 0}$

* You can (and <u>SHOULD</u>) justify this to yourself computationally!

11. a) <u>True</u>; $\vec{u}\times\vec{v}$ is orthogonal to both $\vec{u}$ and $\vec{v}$, so $\vec{u}\cdot(\vec{u}\times\vec{v}) = 0$

b) <u>False</u>; if $\vec{u}$ and $\vec{v}$ are not parallel, $\text{proj}_{\vec{v}}\vec{u}$ is in the direction of $\vec{u}$ while $\text{proj}_{\vec{u}}\vec{v}$ is in the direction of $\vec{v}$

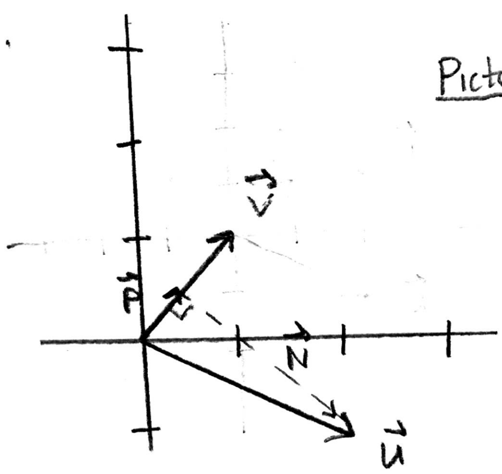
c) <u>False</u>; If $\vec{u},\vec{v}$ are non-zero and parallel, $\theta = 0$ so $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos 0 \neq 0$

d) <u>True</u>; If $\vec{u},\vec{v}$ are parallel, $\theta = 0$, so $|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin 0 = 0$. Hence, $\vec{u}\times\vec{v} = 0$

e) <u>True</u>; If $\vec{u}\perp\vec{v}$, $\theta = \frac{\pi}{2}$ so $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\frac{\pi}{2} = 0$.

f) <u>False</u>; If $\vec{u}\perp\vec{v}$ are both non-zero, $|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\frac{\pi}{2} = |\vec{u}||\vec{v}| \neq 0$.

12.



<u>Pictorially</u>: $\vec{P} = \text{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v}$

$$= \frac{2(1)+(-1)(1)}{(1)(1)+(1)(1)}(\hat{\imath}+\hat{\jmath})$$

$$\boxed{\vec{P} = \frac{1}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}}$$

Since $\vec{u} = \vec{P}+\vec{N}$, $\vec{N} = \vec{u} - \vec{P}$

$$= (2\hat{\imath} - \hat{\jmath}) - \left(\tfrac{1}{2}\hat{\imath} + \tfrac{1}{2}\hat{\jmath}\right)$$

Note that the expressions for $\vec{P}, \vec{N}$ match the picture! $\boxed{\vec{N} = \frac{3}{2}\hat{\imath} - \frac{3}{2}\hat{\jmath}}$

Checking $\vec{N}\perp\vec{v}$: $\vec{N}\cdot\vec{v} = \frac{3}{2}(1) + (-\frac{3}{2})(1) = 0$ ✓

13. • $\vec{u}\times\vec{v}$ is perpendicular to $\vec{u}$ and $\vec{v}$.

• $\vec{u}\times\vec{v} = \det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -7 & 3 \\ 1 & 0 & -3 \end{vmatrix} = [(-7)(-3) - 0(3)]\hat{\imath} - [2(-3) - 1(3)]\hat{\jmath} + [(2)(0) - (1)(-7)]\hat{k}$

$= \underline{21\hat{\imath} + 9\hat{\jmath} + 7\hat{k}}$

• $|\vec{u}\times\vec{v}| = \sqrt{21^2 + 9^2 + 7^2} = \sqrt{571}$

• So, a unit vector $\perp$ to both $\vec{u},\vec{v}$ is $\dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|} = \boxed{\dfrac{1}{\sqrt{571}}[21\hat{\imath} + 9\hat{\jmath} + 7\hat{k}]}$

• Checking: $\vec{u}\cdot\dfrac{(\vec{u}\times\vec{v})}{|\vec{u}\times\vec{v}|} = \dfrac{(2)(21) + (-7)(9) + (3)(7)}{\sqrt{571}} = \dfrac{42 - 63 + 21}{\sqrt{571}} = 0$

$\vec{v}\cdot\dfrac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|} = \dfrac{(21) - (3)(7)}{\sqrt{571}} = 0$ ✓

14. A vector on the line is $\vec{v} = \langle 3-1, 4-2, 0-(-1)\rangle = \langle 2,2,1\rangle$

An equation of the line is: $\vec{r}(t) = \vec{v}t + \vec{P}_0$

$= \langle 2,2,1\rangle t + \langle 1,2,-1\rangle$.

$= \boxed{\langle 2t+1, 2t+2, t-1\rangle}$

15. An equation is $\vec{r}(t) = \vec{v}t + \vec{P}_0 = \langle 1,1,-3\rangle t + \langle 1,0,1\rangle$

$= \boxed{\langle t+1, t, -3t+1\rangle}$

16. • A vector $\perp$ to both $\vec{u},\vec{v}$ is $\vec{u}\times\vec{v}$.

• $\vec{u}\times\vec{v} = \det\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = [(0)(-1) - (1)(1)]\hat{\imath} - [(1)(-1) - (0)(1)]\hat{\jmath} + [(1)(1) - (0)(0)]\hat{k}$

$= -\hat{\imath} + \hat{\jmath} + \hat{k}$

• Thus the line can be described via:

$\vec{r}(t) = \vec{v}t + \vec{P}_0 = \langle -1,1,1\rangle t + \langle -1,2,3\rangle = \boxed{\langle -t-1, t+2, t+3\rangle}$

17a) $\vec{r}(t) = \langle 1+2t, 3-4t, 6+7t \rangle$

$$\boxed{\vec{r}\,'(t) = \langle 2, -4, 7 \rangle}$$

b) $\vec{r}(t) = \langle t^2, 4t^3, 6e^{2t} \rangle$

$$\boxed{\vec{r}\,'(t) = \langle 2t, 12t^2, 12e^{2t} \rangle}$$

c) $\vec{r}(t) = \langle t^2, \cos 2t, \sin t^3 \rangle$

$$\boxed{\vec{r}\,'(t) = \langle 2t, -2\sin 2t, 3t^2 \cos t^3 \rangle}$$

18a) $\vec{v}(t) = \int \vec{a}(t)\,dt = \int \langle -3, 0, -6t \rangle\,dt$

$\vec{v}(t) = \langle -3t + C_1, C_2, -3t^2 + C_3 \rangle$

$\vec{v}(0) = \langle 60, 10, 75 \rangle = \langle C_1, C_2, C_3 \rangle$

So: $\boxed{\vec{v}(t) = \langle -3t + 60, 10, -6t + 75 \rangle}$

b) Speed $= |\vec{v}(t)| = \sqrt{(-3t+60)^2 + 10^2 + (-6t+75)^2}$

$= \sqrt{9t^2 - 360t + 3600 + 100 + 36t^2 - 900t + 5625}$

$$\boxed{|\vec{v}(t)| = \sqrt{45t^2 - 1260t + 9325}}$$

c) $\vec{r}(t) = \int \vec{v}(t)\,dt = \int \langle -3t + 60, 10, -6t + 75 \rangle\,dt$

$= \langle -\frac{3}{2}t^2 + 60t + C_1, 10t + C_2, -3t^2 + 75t + C_3 \rangle$

$\vec{r}(0) = \langle 0, 0, 0 \rangle = \langle C_1, C_2, C_3 \rangle$

So: $\boxed{\vec{r}(t) = \langle -\frac{3}{2}t^2 + 60t, 10t, -3t^2 + 75t \rangle}$

d) The max height is characterized by $\frac{dz}{dt} = 0$

We found in a) $\frac{dz}{dt} = -6t+75$

$$0 = -6t+75$$

$$\underline{t = \frac{25}{2}}$$

Hence, the max height is $z\left(\frac{25}{2}\right) = -3\left(\frac{25}{2}\right)^2 + 75\left(\frac{25}{2}\right)$

$$= \boxed{468.75}$$

e) • We must find when $z(t) = 0$ first:

$$z(t) = -3t^2 + 75t = 0$$

$$-3t(t-25) = 0 \Rightarrow t = 0 \text{ and } \underline{t = 25}$$

↑ starting height!

• $\vec{r}(25) = \left\langle -\frac{3}{2}(25)^2 + 60(25),\ 10(25),\ -3(25)^2 + 75(25) \right\rangle$

$$= \left\langle \frac{1125}{2},\ 250,\ 0 \right\rangle$$

• $|\vec{r}(25)| = \sqrt{\left(\frac{1125}{2}\right)^2 + (250)^2} = \sqrt{\frac{1515625}{4}} = \boxed{\frac{125}{2}\sqrt{97}}$

19. a) $\vec{u}(t) \cdot \vec{v}(t) = t^2(6t) + 3t(4t^2) + 1(e^t)$

$$= \boxed{6t^3 + 12t^3 + e^t}$$

b) $\frac{d}{dt}\left[t^2 \vec{u}(t)\right] = \frac{d}{dt}\left\langle t^4, 3t^3, t^2 \right\rangle = \boxed{\left\langle 4t^3, 9t^2, 2t \right\rangle}$

c) $\vec{u}(t) \times \vec{v}(t) = \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ t^2 & 3t & 1 \\ 6t & 4t^2 & e^t \end{vmatrix} = (3te^t - 4t^2)\hat{i} - (t^2e^t - 6t)\hat{j} + (4t^4 - 9t^2)\hat{k}$

So: $\frac{d}{dt}\left[\vec{u}(t) \times \vec{v}(t)\right] = \left\langle 3e^t + 3te^t - 8t,\ -2te^t - t^2e^t - 6,\ 16t^3 - 18t \right\rangle$

d) $\text{proj}_{\vec{u}(0)} \vec{v}(t) = \text{proj}_{\langle 0,0,1\rangle} \langle 6t, 4t^2, e^t\rangle = \boxed{\langle 0, 0, e^t\rangle}$

This is just the z-component of $\vec{v}(t)$! (CHECK this computationally for yourself! This should be clear both geometrically and computationally)

↑ i.e. NO computation.

e) $\vec{u}(1) = \langle 1, 3, 1\rangle$ so $\vec{u}(1)\cdot\vec{v}(t) = 1(6t) + 3(4t^2) + 1(e^t)$

$= 6t + 12t^2 + e^t$.

So: $\frac{d}{dt}\left(\vec{u}(1)\cdot\vec{v}(t)\right) = \boxed{6 + 24t + e^t}$

f) We found in d) that $\text{proj}_{\vec{u}(0)}\vec{v}(t) = \langle 0, 0, e^t\rangle$.

so: $\frac{d}{dt}\left[\text{proj}_{\vec{u}(0)} \mathbf{v}(t)\right] = \frac{d}{dt}\langle 0, 0, e^t\rangle = \langle 0, 0, e^t\rangle$.

When $t = \ln 3$: $\frac{d}{dt}\left[\text{proj}_{\vec{u}(0)} \mathbf{v}(t)\right] = \boxed{\langle 0, 0, 3\rangle}$