Worksheet #10

I. Basic Vector Problems

1. Given $\vec{u} = 3\hat{i} - \hat{j} + \hat{k}$, $\vec{v} = -\hat{i} + 2\hat{k}$, $\vec{w} = 6\hat{i} + \hat{j} - \hat{k}$. Find:

   a) $2\vec{u} - 3\vec{v}$

   b) $|\vec{u}|$

   c) $|\vec{v} + 2\vec{w}|$

   d) A unit vector in the direction of $\vec{v}$.

   e) A vector parallel to $\vec{u} + 2\vec{w}$ with magnitude 2.

   f) A vector $\vec{a}$ parallel to $\vec{v}$ whose magnitude is $|\vec{w}|$.

2. Find all vectors $\vec{u}$ that are equal in magnitude to $\vec{v} = -\hat{i} + \hat{j}$ and parallel to $\vec{w} = \hat{j} + \hat{k}$.

3. True or False

   a) If $\vec{u} = \vec{v}$, then $|\vec{u}| = |\vec{v}|$

   b) If $|\vec{u}| = |\vec{v}|$, then $\vec{u} = \vec{v}$.

   c) For any real number c, $|c\vec{u}| = c|\vec{u}|$.

   d) Two vectors $\vec{u} = \langle u_1, u_2, u_3 \rangle$ and $\vec{v} = \langle v_1, v_2, v_3 \rangle$ are equal iff $u_1 = v_1$, $u_2 = v_2$, and $u_3 = v_3$.

II. Dot Products

4. Given $\vec{u} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{v} = -\hat{i} + 2\hat{j} - \hat{k}$, find:

   a) $\vec{u} \cdot \vec{v}$

   b) $2\vec{u} \cdot 3\vec{v}$

   c) $\vec{u} \cdot (\vec{u} + \vec{v})$

   d) The angle between $\vec{u}$ and $\vec{v}$.

   e) $\text{scal}_{\vec{v}}\, \vec{u}$

   f) $\vec{u} \cdot \text{proj}_{\vec{v}}\, \vec{u}$

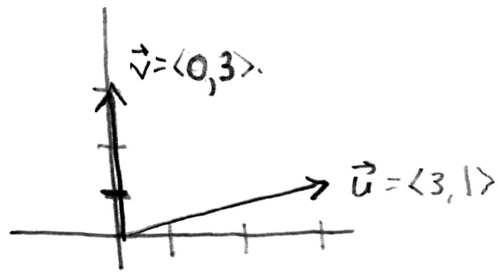
5. For any vector $\vec{u}$, find $\text{proj}_{\vec{u}}\, \vec{u}$.

6. a) Are $\vec{u} = \langle -1, 3 \rangle$ and $\vec{v} = \langle 2, 1 \rangle$ orthogonal?

   b) Find a description of all vectors orthogonal to $\vec{u}$.

   c) Justify this geometrically!

7. Indicate on the following picture what the vector $\text{proj}_{\vec{v}}\vec{u}$ is. Then compute it:



8. Show that $\text{proj}_{\vec{v}}\,\vec{u} = \text{proj}_{\vec{w}}\,\vec{u}$ iff $\vec{v}$ and $\vec{w}$ are parallel.

III. Cross-Products

9. Given $\vec{u}= 2\hat{\imath}-3\hat{\jmath}+\hat{k}$, $\vec{v}= \hat{\imath}+\hat{\jmath}-\hat{k}$; find

a) $\vec{u}\times\vec{v}$

b) A vector orthogonal to both $\vec{u}$ and $\vec{v}$.

c) $\text{proj}_{\vec{u}}(\vec{u}\times\vec{v})$

d) The area of the parallelogram generated by $\vec{u}$ and $\vec{v}$.

10. Given $\vec{u}= 3\hat{\imath}-\hat{\jmath}$, $\vec{v}= 2\hat{\jmath}+\hat{k}$, $\vec{w}= \hat{\imath}+\hat{\jmath}+\hat{k}$, find

a) $(\vec{u}\times\vec{v})\times\vec{w}$

b) $\vec{u}\times(\vec{v}\times\vec{w})$

c) $\vec{u}\cdot(\vec{v}\times\vec{w})$.

d) $\text{proj}_{\vec{w}}\,\vec{u}\times\vec{v}$

e) A vector orthogonal to $\vec{u}$ and $\vec{v}$.

f) $\text{proj}_{3\vec{v}}(\vec{v}\times\vec{w})$

IV. Questions Involving Dot and Cross Products

11. True or False

a) For any vectors $\vec{u}$ and $\vec{v}$, $\vec{u}\cdot(\vec{u}\times\vec{v})=0$.

b) For any vectors $\vec{u}$ and $\vec{v}$, $\text{proj}_{\vec{v}}\vec{u}=\text{proj}_{\vec{u}}\vec{v}$.

c) If $\vec{u}$ and $\vec{v}$ are parallel, $\vec{u}\cdot\vec{v}=0$.

d) If $\vec{u}$ and $\vec{v}$ are parallel, $\vec{u}\times\vec{v}=0$.

e) If $\vec{u}$ and $\vec{v}$ are orthogonal, $\vec{u}\cdot\vec{v}=0$

f) If $\vec{u}$ and $\vec{v}$ are orthogonal, $\vec{u}\times\vec{v}=0$

12. Let $\vec{u}= 2\hat{\imath}-\hat{\jmath}$ and $\vec{v}= \hat{\imath}+\hat{\jmath}$. Find a vector $\vec{P}$ that is parallel to $\vec{v}$ and a vector $\vec{N}$ that is perpendicular to $\vec{v}$ so $\vec{u}=\vec{P}+\vec{N}$. Check explicitly that $\vec{N}\perp\vec{v}$.

13. Given $\vec{u} = 2\hat{i} - 7\hat{j} + 3\hat{k}$, $\vec{v} = \hat{i} - 3\hat{k}$, find a unit vector perpendicular to both $\vec{u}$ and $\vec{v}$. Check explicitly that this vector is perpendicular to them.

V. Vector-Valued Functions

14. Find an equation of the line passing through $(1,2,-1)$ and $(3,4,0)$.

15. Find an equation for the line parallel to $\vec{v} = \hat{i} + \hat{j} - 3\hat{k}$ that passes through $(1,0,1)$.

16. Find the equation of the line that is perpendicular to both $\vec{u} = \hat{i} + \hat{k}$ and $\vec{v} = \hat{j} - \hat{k}$ that passes through $(-1,2,3)$.

17. Find $\vec{r}\,'(t)$ for the following curves.

a) $\vec{r}(t) = \langle 1+2t,\ 3-4t,\ 6+7t \rangle$

b) $\vec{r}(t) = \langle t^2,\ 4t^3,\ 6e^{2t} \rangle$.

c) $\vec{r}(t) = \langle t^2,\ \cos 2t,\ \sin t^3 \rangle$.

18. A particle starts $\langle 0,0,0 \rangle$ with has acceleration $\langle -3, 0, -6t \rangle$ and initial velocity $\vec{v}(0) = \langle 60, 10, 75 \rangle$. Find:

a) The velocity function $\vec{v}(t)$.

b) The speed function.

c) The position function.

d) The maximum height of the particle

e) The total distance the ball is from its starting location once it hits the ground.

19. Given $\vec{u}(t) = \langle t^2, 3t, 1 \rangle$, $\vec{v}(t) = \langle 6t, 4t^2, e^t \rangle$, find:

a) $\vec{u}(t) \cdot \vec{v}(t)$

b) $\frac{d}{dt}\left[t^2\, \vec{u}(t) \right]$.

c) $\frac{d}{dt}\left[\vec{u}(t) \times \vec{v}(t) \right]$

d) The vector projection of $\vec{v}(t)$ onto $\vec{u}(0)$.

e) $\frac{d}{dt}\left[\vec{u}(1) \cdot \vec{v}(t) \right]$.

f) $\frac{d}{dt}\left[\mathrm{proj}_{u(0)}\, \vec{v}(t) \right]$ at $t = \ln 3$.