Worksheet # 4 Answers

1. $-\frac{1}{12}\cos 4x^3 + C$
2. $\frac{2}{3}\ln|3x^2+9| + C$
3. $\frac{5}{882}(14x^9-1)^7 + C$
4. $\frac{3}{4}(12x+3x^2)^{2/3} + C$
5. $\frac{1}{12}(e^5 - e)$
6. $\sin\sqrt{\pi/2} - \sin\sqrt{\pi/4}$
7. $\frac{1}{2}\ln 8 - \frac{1}{2}\ln 4$ -OR- $\frac{1}{2}\ln 2$
8. 0
9. $\frac{1}{8}x^8 + \frac{2}{5}x^5 + \frac{1}{2}x^2 + C$
10. $-\frac{5}{3}x^{-1} - \frac{5}{3}\ln|x| + C$
11. $\frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{11}{2}x - \frac{13}{4}\ln|2x+1| + C$
12. $\frac{1}{6}\ln|9x^2 + 16x^{3/2}| + C$
13. $\frac{5}{2}x^2 - 5x + 6\ln|x+1| + C$
14. $\ln|\ln x| + C$
15. $\frac{1}{18}(x^3+1)^6 + C$
16. $14\tan e^x + C$
17. $\frac{4}{5}x\sin 5x + \frac{4}{25}\cos 5x + C$
18. $x\arcsin x + \sqrt{1-x^2} + C$
19. $x(\ln x)^2 - 2x\ln x + 2x + C$
20. $-\frac{1}{4}x^2\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + C$
21. $\frac{1}{4}e^2 + \frac{1}{4}$
22. $\frac{1}{10}e^{2\pi} - \frac{1}{10}$
23. $-4/5$

24. OMIT

25. $-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C$

26. $\sec^5 x - \frac{5}{3}\sec^3 x + C$

27. $-\frac{1}{2}\ln|\cos 2x| + \frac{1}{4}\cos^2 2x + C$

28. $\frac{1}{8}x - \frac{1}{96}\sin 12x + C$

29. $44/15$

30. $544/81$

31. $3/8$

32. $\frac{3}{2}\sqrt{2} - \frac{5}{2}$

33. $\frac{1}{16}\arcsin 2x - \frac{1}{4}\frac{x}{\sqrt{1-4x^2}} + C$

34. $\frac{3}{4}\ln\left|\frac{\sqrt{9+16x^2}}{3} + \frac{4x}{3}\right| + C$

35. $\frac{5}{3}\frac{\sqrt{25-9x^2}}{x} - 5\operatorname{arcsec}\frac{3x}{5} + C$

36. $\frac{8x}{\sqrt{1+36x^2}} + C$

37. $-\frac{1}{4}\frac{\sqrt{4-x^2}}{x} + C$

38. $-\frac{3}{40}\ln\left|\frac{5x}{\sqrt{25x^2-16}} + \frac{4}{\sqrt{25x^2-16}}\right| + C$

39. ⟨See solutions⟩ This is a good question!

Worksheet # 4 Solutions

1. $\int 3x^2 \sin 4x^3\,dx = \int 3\cancel{x^2} \sin u \cdot \frac{1}{12}\frac{1}{\cancel{x^2}}\,du$

$u = 4x^3$

$du = 12x^2\,dx$

$dx = \frac{1}{12}\frac{1}{x^2}\,du$

$= -\frac{1}{12}\cos u + C$

$= \boxed{-\frac{1}{12}\cos 4x^3 + C}$

2. $\int \frac{4x+6}{3x^2+9x}\,dx = \int \frac{4x+6}{u} \cdot \frac{du}{6x+9}$

$u = 3x^2+9x$

$du = (6x+9)\,dx$

$\frac{du}{6x+9} = dx$

$= \int \frac{2\cancel{(2x+3)}}{u} \cdot \frac{du}{3\cancel{(2x+3)}}$

$= \frac{2}{3}\ln|u| + C$

$= \boxed{\frac{2}{3}\ln|3x^2+9x| + C}$

3. $\int 5x^8(14x^9-1)^6\,dx = \int 5\cancel{x^8}\,u^6 \cdot \frac{du}{126\cancel{x^8}}$

$u = 14x^9-1$

$du = 126x^8\,dx$

$\frac{du}{126x^8} = dx$

$= \frac{5}{126} \cdot \frac{1}{7}u^7 + C$

$= \boxed{\frac{5}{882}(14x^9-1)^7 + C}$

4. $\int \frac{6+3x}{\sqrt[3]{12x+3x^2}}\,dx = \int \frac{6+3x}{\sqrt[3]{u}}\,\frac{du}{12+6x}$

$u = 12x+3x^2$

$du = (12+6x)\,dx$

$\frac{du}{12+6x} = dx$

$= \int \frac{3\cancel{(2+x)}}{\sqrt[3]{u}} \cdot \frac{du}{6\cancel{(2+x)}}$

$= \int \frac{1}{2}u^{-1/3}\,du$

$= \frac{1}{2}\cdot\frac{3}{2}u^{2/3} + C$

$= \boxed{\frac{3}{4}(12x+3x^2)^{2/3} + C}$

5. $\int_0^1 x^2 e^{4x^3+1}\,dx = \int_{x=0}^{x=1} \cancel{x^2} e^u \cdot \frac{du}{12\cancel{x^2}}$

$u = 4x^3+1$

$du = 12x^2\,dx$

$\frac{du}{12x^2} = dx$

$= \frac{1}{12} e^u \Big|_{x=0}^{x=1}$

$= \frac{1}{12} e^{4x^3+1} \Big|_0^1$

$= \frac{1}{12} e^5 - \frac{1}{12} e^1$

$= \boxed{\frac{1}{12}(e^5 - e)}$

– OR –

$u = 4x^3+1 \rightarrow$ $u=5$ when $x=1$

$u=1$ when $x=0$

so $\int_0^1 x^2 e^{4x^3+1}\,dx$

$= \int_{u=1}^{u=5} \frac{1}{12} e^u\,du$

$= \frac{1}{2} e^u \Big|_1^5$

6. $\int_{\pi/4}^{\pi/2} \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx = \int_{x=\pi/4}^{x=\pi/2} \frac{\cos u}{\cancel{2\sqrt{x}}} \cdot \cancel{2x^{1/2}}\,du$

Let $u = x^{1/2}$

$du = \frac{1}{2} x^{-1/2}\,dx$

$2x^{1/2}\,du = dx$

$= \sin u \Big|_{x=\pi/4}^{x=\pi/2}$

$= \boxed{\sin\sqrt{\frac{\pi}{2}} - \sin\sqrt{\frac{\pi}{4}}}$

7. $\int_0^2 \frac{1}{2x+4}\,dx = \int_{x=0}^{x=2} \frac{1}{u} \cdot \frac{1}{2}\,du$

$u = 2x+4$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$= \frac{1}{2} \ln|u| \Big|_{x=0}^{x=2}$

$= \frac{1}{2} \ln|2x+4| \Big|_0^2$

$= \boxed{\frac{1}{2}\ln 8 - \frac{1}{2}\ln 4}$ or $\boxed{\frac{1}{2}\ln 2}$

8. $\int_{\sqrt{\pi/6}}^{\sqrt{\pi/3}} 2x \sec 4x^2 \tan 4x^2\,dx = \int_{x=\sqrt{\pi/6}}^{x=\sqrt{\pi/3}} 2\cancel{x} \sec u \tan u \cdot \frac{du}{8\cancel{x}}$

$u = 4x^2$

$du = 8x\,dx$

$\frac{du}{8x} = dx$

$= \frac{1}{4} \sec u \Big|_{x=\sqrt{\pi/6}}^{x=\sqrt{\pi/3}}$

$= \frac{1}{4} \sec 4x^2 \Big|_{\sqrt{\pi/6}}^{\sqrt{\pi/3}}$

$= \frac{1}{4} \sec\frac{4\pi}{3} - \frac{1}{4} \sec\frac{2\pi}{3} = \boxed{0}$

II.

9. $$\int x(x^3+1)^2\,dx = \int x\,(x^6+2x^3+1)\,dx$$
$$= \int (x^7+2x^4+x)\,dx$$
$$= \boxed{\frac{1}{8}x^8+\frac{2}{5}x^5+\frac{1}{2}x^2+C}$$

10. $$\int \frac{5x-5x^2}{3x^3}\,dx = \int\left(\frac{5}{3}\frac{x}{x^3}-\frac{5}{3}\frac{x^2}{x^3}\right)dx$$
$$= \int\left(\frac{5}{3}x^{-2}-\frac{5}{3}\frac{1}{x}\right)dx$$
$$= \boxed{-\frac{5}{3}x^{-1}-\frac{5}{3}\ln|x|+C}$$

11. $$\int \frac{2x^3-4x^2+8x-1}{2x+1}\,dx$$ ← deg numerator > deg denom

$$\begin{array}{r l}
 & x^2-3x+\frac{11}{2}-\frac{13}{2}\frac{1}{2x+1} \\
2x+1 \big) & 2x^3-4x^2+8x-1 \\
 & -(2x^3+2x^2) \downarrow \\
 & -6x^2+8x \\
 & -(-6x^2-3x) \\
 & 11x-1 \\
 & -\left(11x+\frac{11}{2}\right) \\
 & -\frac{13}{2}
\end{array}$$

So: $$\int \frac{2x^3-4x^2+8x-1}{2x+1}\,dx = \int\left(x^2-3x+\frac{11}{2}-\frac{13}{2}\,\frac{1}{2x+1}\right)dx$$
$$= \boxed{\frac{1}{3}x^3-\frac{3}{2}x^2+\frac{11}{2}x-\frac{13}{4}\ln|2x+1|+C}$$

12. $$\int \frac{3x+4\sqrt{x}}{9x^2+16\sqrt{x^3}}\,dx = \int \frac{3x+4x^{1/2}}{u}\,\frac{du}{18x+24x^{1/2}}$$

Let $u = 9x^2+16x^{3/2}$

$du = (18x+24x^{1/2})\,dx$

$\dfrac{du}{(18x+24x^{1/2})} = dx$

$$= \int \frac{\cancel{3x+4x^{1/2}}}{u}\cdot\frac{du}{6(\cancel{3x+4x^{1/2}})}$$
$$= \frac{1}{6}\ln|u|+C = \boxed{\frac{1}{6}\ln\left|9x^2+16x^{3/2}\right|+C}$$

13. $\displaystyle\int \frac{5x^2+1}{x+1}\,dx = \int 5x-5+\frac{6}{x+1}\,dx$

$$\begin{array}{r} 5x - 5 + \frac{6}{x+1} \\ x+1 \overline{)\, 5x^2 + 0x + 1} \\ -(5x^2+5x) \\ \hline -5x+1 \\ (-5x-5) \\ \hline 6 \end{array}$$

$= \boxed{\frac{5}{2}x^2 - 5x + 6\ln|x+1| + C}$

14. $\displaystyle\int \frac{1}{x\ln x}\,dx = \int \frac{1}{\cancel{x}}\frac{1}{u}\cdot \cancel{x}\,du$

What else could we do? → $u = \ln x$

Let's hope this works!

$du = \frac{1}{x}\,dx$

$x\,du = dx$

$= \ln|u| + C$

$= \boxed{\ln|\ln x| + C}$

15. $\displaystyle\int x^2(x^3+1)^5\,dx = \int x^2 u^5 \frac{du}{3x^2}$

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$= \frac{1}{3}\cdot\frac{1}{6}u^6 + C$

$= \boxed{\frac{1}{18}(x^3+1)^6 + C}$

16. $\displaystyle\int 14e^x\sec^2(e^x)\,dx = \int 14\cancel{e^x}\sec^2 u\,\frac{du}{\cancel{e^x}}$

$u = e^x$

$du = e^x\,dx$

$\frac{du}{e^x} = du$

$= 14\tan u + C$

$= \boxed{14\tan e^x + C}$

17. $\displaystyle\int 4x\cos 5x\,dx$

$u = 4x \qquad dv = \cos 5x\,dx$

$du = 4\,dx \qquad v = \frac{1}{5}\sin 5x$

$u \quad v \quad - \int v \quad du$

$= (4x)(\frac{1}{5}\sin 5x) - \int \frac{4}{5}\sin 5x\,du$

$= \boxed{\frac{4}{5}x\sin 5x + \frac{4}{25}\cos 5x + C}$

18. $\int \arcsin x \, dx$ $= x \arcsin x - \int \frac{x}{\sqrt{1-x^2}} \, dx$

$u = \arcsin x \quad dv = dx$

$du = \frac{1}{\sqrt{1-x^2}} \, dx \quad v = x$

$u = 1-x^2$

$du = -2x \, dx$

$\frac{du}{-2x} = dx$

$= x \arcsin x - \int \frac{x}{\sqrt{u}} \frac{du}{-2x}$

$= x \arcsin x + \frac{1}{2} \int u^{-1/2} \, du$

$= x \arcsin x + \frac{1}{2} \cdot 2u^{1/2} + C$

$= \boxed{x \arcsin x + \sqrt{1-x^2} + C}$

19. $\int (\ln x)^2 \, dx$ $= x(\ln x)^2 - \int x \cdot 2\ln x \cdot \frac{1}{x} \, dx$

$u = (\ln x)^2 \quad dv = dx$

$du = 2\ln x \cdot \frac{1}{x} \, dx \quad v = x$

$= x(\ln x)^2 - 2\int \ln x \, dx$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x} \, dx \quad v = x$

$= x(\ln x)^2 - 2\left[x\ln x - \int x \cdot \frac{1}{x} \, dx \right]$

$= \boxed{x(\ln x)^2 - 2x\ln x + 2x + C}$

suggest 2 int by parts needed!

20. $\int x^2 \sin 4x \, dx$ $= -\frac{1}{4}x^2 \cos 4x - \int -\frac{1}{4}\cos 4x \cdot 2x \, dx$

$u = x^2 \quad dv = \sin 4x \, dx$

$du = 2x \, dx \quad v = -\frac{1}{4}\cos 4x$

$= -\frac{1}{4}x^2 \cos 4x + \frac{1}{2}\int x \cos 4x \, dx$

$u = x \quad dv = \cos 4x \, dx$

$du = dx \quad v = \frac{1}{4}\sin 4x$

$= -\frac{1}{4}x^2 \cos 4x + \frac{1}{2}\left[\frac{1}{4}x \sin 4x - \int \frac{1}{4}\sin 4x \, dx \right]$

$= \boxed{-\frac{1}{4}x^2 \cos 4x + \frac{1}{8}x \sin 4x + \frac{1}{32}\cos 4x + C}$

21. $\int_1^e x \ln x \, dx$ $= \frac{1}{2}x^2 \ln x \Big|_1^e - \int_1^e \frac{1}{2}x^2 \cdot \frac{1}{x} \, dx$

$u = \ln x \quad dv = x \, dx$

$du = \frac{1}{x} \, dx \quad v = \frac{1}{2}x^2$

$= \frac{1}{2}e^2 \ln e - \cancel{\frac{1}{2}(1)^2 \ln 1}^{\,0} - \frac{1}{2}\int_1^e x \, dx$

$= \frac{1}{2}e^2 - \frac{1}{2}\left[\frac{1}{2}x^2\Big|_1^e\right] = \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \boxed{\frac{1}{4}e^2 + \frac{1}{4}}$

22. $\int_0^{\pi} e^{2x} \cos 4x \, dx := I$

$u = \cos 4x \qquad dv = e^{2x}$

$du = -4 \sin 4x \, dx \qquad v = \frac{1}{2} e^{2x}$

$$I = \frac{1}{2} e^{2x} \cos 4x \Big|_0^{\pi} - \int_0^{\pi} \frac{1}{2} e^{2x} (-4 \sin 4x) \, dx$$

$$= \frac{1}{2} e^{2\pi} \cos 4\pi - \frac{1}{2} e^{0} \cos 0 + \int 2e^{2x} \sin 4x$$

$$= \frac{1}{2} e^{2\pi} - \frac{1}{2}$$

$u = \sin 4x \qquad dv = 2e^{2x}$

$du = 4 \cos 4x \qquad v = e^{2x}$

$$= \frac{1}{2} e^{2\pi} - \frac{1}{2} + \left[e^{2x} \sin 4x \Big|_0^{\pi} - \int_0^{\pi} 4e^{2x} \cos 4x \, dx \right]$$

$$I = \frac{1}{2} e^{2\pi} - \frac{1}{2} + e^{2\pi} \overset{0}{\cancel{\sin 4\pi}} - e^{0} \overset{0}{\cancel{\sin 0}} - 4 \underbrace{\int_0^{\pi} e^{2x} \cos 4x \, dx}_{= I!}$$

$$I = \frac{1}{2} e^{2\pi} - \frac{1}{2} - 4I$$

$$5I = \frac{1}{2} e^{2\pi} - \frac{1}{2}$$

$$\boxed{I = \frac{1}{10} e^{2\pi} - \frac{1}{10}}$$

23. $I = \int_0^{\pi} \sin 2x \cos 3x \, dx$ ← Trig sub won't work because the angles are different!

$u = \sin 2x \qquad dv = \cos 3x \, dx$

$du = 2 \cos 2x \, dx \qquad v = \frac{1}{3} \sin 3x$

$$I = \frac{1}{3} \sin 2x \sin 3x \Big|_0^{\pi} - \int \frac{2}{3} \cos 2x \sin 3x \, dx$$

$$= \frac{1}{3} \overset{0}{\cancel{\sin 2\pi}} \overset{0}{\sin 3\pi} - \frac{1}{3} \overset{0}{\cancel{\sin 0}} \overset{0}{\cancel{\sin 0}} - \frac{2}{3} \int_0^{\pi} \cos 2x \sin 3x \, dx$$

$u = \cos 2x \qquad dv = \sin 3x \, dx$

$du = -2 \sin 2x \, dx \qquad v = -\frac{1}{3} \cos 3x$

$$I = -\frac{2}{3} \left[-\frac{1}{3} \cos 2x \cos 3x \Big|_0^{\pi} - \int_0^{\pi} -\frac{1}{3} \cos 3x (-2 \sin 2x) dx \right]$$

I!

$$I = -\frac{2}{3} \left[-\frac{1}{3} \cos 2\pi \cos 3\pi + \frac{1}{3} \cos 0 \cos 0 - \frac{2}{3} \int_0^{\pi} \sin 2x \cos 3x \, dx \right].$$

$$I = -\frac{2}{3}\left[-\frac{1}{3}(1)(-1) + \frac{1}{3} - \frac{2}{3}I\right]$$

$$I = -\frac{4}{9} + \frac{4}{9}I$$

$$\frac{5}{9}I = -\frac{4}{9} \Rightarrow \boxed{I = -4/5}$$

24. OMIT

IV.

25. $\int \sin^3 x \cos^2 x \, dx = \int \sin^3 x \; u^2 \frac{du}{-\sin x}$

$u = \cos x$

$du = -\sin x \, dx$

$\frac{du}{-\sin x} = dx$

$$= -\int \sin^2 x \, u^2 \, du$$

$$= -\int (1 - \cos^2 x) \, u^2 \, du$$

$$= -\int (1 - u^2) \, u^2 \, du$$

$$= \int (-u^2 + u^4) \, du$$

$$= -\frac{1}{3}u^3 + \frac{1}{5}u^5 + C$$

$$= \boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + C}$$

26. $\int 5 \sec^3 x \tan^3 x \, dx = \int 5 \sec^2 x \tan^2 x \, (\sec x \tan x) \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

↑ save for dx!

$\frac{du}{\sec x \tan x} = dx$

$$= \int 5u^2(\sec^2 x - 1) \cancel{\sec x \tan x} \frac{du}{\cancel{\sec x \tan x}}$$

$$= \int 5u^2(u^2 - 1) \, du$$

$$= \int (5u^4 - 5u^2) \, du$$

$$= u^5 - \frac{5}{3}u^3 + C$$

$$= \boxed{\sec^5 x - \frac{5}{3}\sec^3 x + C}$$

27. $\int \frac{\sin^3 2x}{\cos 2x} \, dx = \int \frac{\sin^{\cancel{3}\,2} 2x}{u} \frac{du}{-2\cancel{\sin 2x}}$

$u = \cos 2x$

$du = -2 \sin 2x \, dx$

$\frac{du}{-2\sin 2x} = dx$

$= \int -\frac{1}{2} \frac{1 - \cos^2 2x}{u}$

$= \int -\frac{1}{2} \frac{1-u^2}{u} \, du$

$= -\frac{1}{2} \int \left(\frac{1}{u} - u \right) du$

$= -\frac{1}{2} \left[\ln|u| - \frac{1}{2} u^2 \right] + C$

$= \boxed{-\frac{1}{2} \ln|\cos 2x| + \frac{1}{4} \cos^2 2x + C}$

28. $\int \sin^2 3x \cos^2 3x \, dx$ ← use identities!

$= \int \left(\frac{1}{2} - \frac{1}{2} \cos 6x \right) \left(\frac{1}{2} + \frac{1}{2} \cos 6x \right) dx$

$= \int \left[\frac{1}{4} - \frac{1}{4} \cos^2 6x \right] dx$

$= \int \left[\frac{1}{4} - \frac{1}{4} \left(\frac{1}{2} + \frac{1}{2} \cos 12x \right) \right] dx$

$= \int \left(\frac{1}{4} - \frac{1}{8} - \frac{1}{8} \cos 12x \right) dx$

$= \int \left(\frac{1}{8} - \frac{1}{8} \cos 12x \right) dx$

$= \boxed{\frac{1}{8} x - \frac{1}{96} \sin 12x + C}$

29. $\int_0^{\pi} (4\sin x + \sin^3 x)\cos^2 x \quad dx$

$= \int_0^{\pi} 4\sin x \cos^2 x \quad dx + \underbrace{\int_0^{\pi} \sin^3 x \cos^2 x \, dx}_{\text{Done in \#25}}$

$u = \cos x$

$du = -\sin x \, dx$

$\frac{du}{-\sin x} = dx$

$= \int_{x=0}^{x=\pi} 4\cancel{\sin x}\, u^2 \frac{du}{-\cancel{\sin x}} + \left[-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x \right|_0^{\pi}$

$= -4 \cdot \frac{1}{3} u^3 \Big|_{x=0}^{x=\pi} + \left[\left(-\frac{1}{3}(-1) + \frac{1}{5}(-1)\right) - \left(-\frac{1}{3} + \frac{1}{5}\right)\right]$

$= -\frac{4}{3}\cos^3 x \Big|_0^{\pi} + \left[\frac{1}{3} - \frac{1}{5} + \frac{1}{3} - \frac{1}{5}\right]$

$= -\frac{4}{3}(-1) - \left(-\frac{4}{3}\right) + \frac{2}{3} - \frac{2}{5} = \boxed{44/15}$

30. $\int_{\pi/6}^{\pi/3} \tan^3 x \sec^4 x \, dx = \int_{x=\pi/6}^{x=\pi/3} u^3 \sec^4 x \frac{du}{\cancel{\sec^2 x}}$

$u = \tan x$

$du = \sec^2 x \, dx$

$\frac{du}{\sec^2 x} = dx$

$= \int_{x=\pi/6}^{x=\pi/3} u^3(\tan^2 x + 1)\, du$

$= \int_{x=\pi/6}^{x=\pi/3} u^3(u^2+1)\, du$

$= \int_{x=\pi/6}^{x=\pi/3} (u^5 + u^3)\, du$

$= \frac{1}{6}u^6 + \frac{1}{4}u^4 \Big|_{x=\pi/6}^{x=\pi/3}$

$= \left[\frac{1}{6}\tan^6 x + \frac{1}{4}\tan^4 x \right|_{\pi/6}^{\pi/3}$

$= \boxed{544/81}$

$(\sin^2 \pi x)^2$

31. $\int_0^1 \sin^4 \pi x \, dx. = \int_0^1 \left(\frac{1}{2} - \frac{1}{2}\cos 2\pi x\right)^2 dx$

$= \int_0^1 \left(\frac{1}{4} - \frac{1}{2}\cos 2\pi x + \frac{1}{4}\cos^2 2\pi x\right) dx$

$= \int_0^1 \left(\frac{1}{4} - \frac{1}{2}\cos 2\pi x + \frac{1}{8} + \frac{1}{8}\cos 4\pi x\right) dx$

$$= \int_0^1 \left(\frac{3}{8} - \frac{1}{2}\cos 2\pi x + \frac{1}{8}\cos 4\pi x\right) dx$$

$$= \left[\frac{3}{8}x - \frac{1}{4\pi}\sin 2\pi x + \frac{1}{32\pi}\sin 4\pi x \right|_0^1$$

$$= \left[\frac{3}{8} - \frac{1}{4\pi}\sin 2\pi + \frac{1}{32\pi}\sin 4\pi\right] - 0 = \boxed{3/8}$$

32. $$\int_{\pi/4}^{\pi/3} \frac{\tan^3 x}{\sec x}\, dx. = \int_{x=\pi}^{x=\pi} \frac{\tan^{\cancel{3}\,2} x}{\sec x} \frac{du}{\sec x \cancel{\tan x}}$$

$u = \sec x$

$du = \sec x \tan x \; dx$

$\frac{du}{\sec x \tan x} = dx$

$$= \int_{x=\pi/4}^{x=\pi/3} \frac{\sec^2 x - 1}{\sec^2 x}\, du$$

$$= \int_{x=\pi/4}^{x=\pi/3} \frac{u^2 - 1}{u^2}\, du$$

$$= \int_{x=\pi/4}^{x=\pi/3} (1 - u^{-2})\, du$$

$$= u + u^{-1} \Big|_{x=\pi/4}^{x=\pi/3}$$

$$= \left[\sec x + \cos x \right|_{x=\pi/4}^{x=\pi/3}$$

$$= \boxed{\frac{3}{2}\sqrt{2} - \frac{5}{2}}$$

33. $$\int \frac{x^2}{\sqrt{1-4x^2}}$$ ← "−" in front of variable

1. $a^2 = 1 \rightarrow a = 1$

$u^2 = 4x^2 \rightarrow u = 2x$

$u = a \sin\theta$

$2x = \sin\theta$

$x = \frac{1}{2}\sin\theta$

$dx = \frac{1}{2}\cos\theta \, d\theta$

2. Substitute and Evaluate.

$$\int \frac{x^2}{\sqrt{1-4x^2}}\, dx = \int \frac{\frac{1}{4}\sin^2\theta}{\sqrt{1-\sin^2\theta}} \frac{1}{2}\cos\theta\, d\theta$$

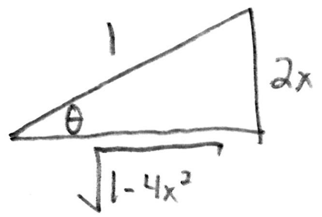
$$= \frac{1}{8}\int \frac{\sin^2\theta}{\cancel{\sqrt{\cos^2\theta}}} \cancel{\cos\theta}\, d\theta$$

$$= \frac{1}{8}\int \left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$$

$$= \frac{1}{8}\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right] + C$$

$$I = \frac{1}{16}\theta - \frac{1}{32}\sin 2\theta + C$$

3. Make the triangle



$\leftarrow \frac{2x}{1} = \sin\theta$

$\theta = \arcsin 2x$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2(2x)\left(\frac{2x}{\sqrt{1-4x^2}}\right)$

$= \frac{8x}{\sqrt{1-4x^2}}$

So $I = \frac{1}{16}\arcsin 2x - \frac{1}{32}\cdot\frac{8x}{\sqrt{1-4x^2}} + C = \boxed{\frac{1}{16}\arcsin 2x - \frac{x}{4\sqrt{1-4x^2}} + C}$

34. $I = \int \frac{3}{\sqrt{9+16x^2}}\,dx.$

1. Find a, u

$a^2 = 9 \rightarrow a = 3$

$u^2 = 16x^2 \rightarrow u = 4x$

$u = a\tan\theta \leftarrow$ No "-"

$4x = 3\tan\theta$

$x = \frac{3}{4}\tan\theta$

$dx = \frac{3}{4}\sec^2\theta\,d\theta$

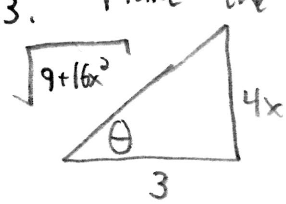
2. Make the substitution:

$I = \int \frac{3}{\sqrt{9+\cancel{16}(\frac{9}{\cancel{16}}\tan^2\theta)}}\,\frac{3}{4}\sec^2\theta\,d\theta$

$= \int \frac{\cancel{3}}{\cancel{3}\sec\theta}\cdot\frac{3}{4}\sec^{\cancel{2}}\theta\,d\theta$

$= \frac{3}{4}\ln|\sec\theta + \tan\theta| + C$

3. Make the triangle



$4x = 3\tan\theta \rightarrow \tan\theta = \frac{4x}{3}$

So $\boxed{I = \frac{3}{4}\ln\left|\frac{\sqrt{9+16x^2}}{3} + \frac{4x}{3}\right| + C}$

35. $I = \int \frac{\sqrt{9x^2 - 25}}{x} dx$

1. $u^2 = 9x^2 \rightarrow u = 3x$

$a^2 = 25 \rightarrow a = 5$

"$-$" is in front of a

$\Rightarrow \quad u = a \sec\theta$

$3x = 5\sec\theta$

$x = \frac{5}{3}\sec\theta$

$dx = \frac{5}{3}\sec\theta\tan\theta\, d\theta$

2. Do the subs:

$I = \int \frac{\sqrt{9(\frac{25}{9}\sec^2\theta) - 25}}{\cancel{5/3}\ \cancel{\sec\theta}} \cdot \cancel{\frac{5}{3}}\ \cancel{\sec\theta}\tan\theta\, d\theta$

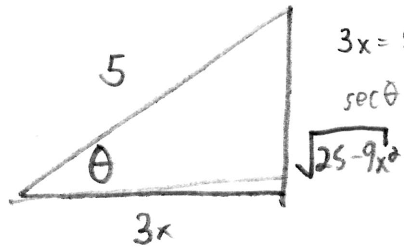
$= \int 5\tan\theta \cdot \tan\theta\, d\theta$

$= 5\int \tan^2\theta\, d\theta$

$= 5\int (\sec^2\theta - 1)\, d\theta$

$= 5\tan\theta - 5\theta + C$

3. Draw the triangle:



$3x = 5\sec\theta$

$\sec\theta = \frac{3x}{5}$

$\Rightarrow I = 5\frac{\sqrt{25 - 9x^2}}{3x} - 5\operatorname{arcsec}\frac{3x}{5} + C$

36. $I = \int \frac{8}{(36x^2 + 1)^{3/2}} dx = \int \frac{8}{(\cancel{36} \cdot \frac{1}{\cancel{36}}\tan^2\theta + 1)^{3/2}} \frac{1}{6}\sec^2\theta\, d\theta$

$u^2 = 36x^2 \rightarrow u = 6x$

$a^2 = 1 \rightarrow a = 1$

No "$-$" $\Rightarrow \quad u = a\tan\theta$

$6x = \tan\theta$

$x = \frac{1}{6}\tan\theta$

$dx = \frac{1}{6}\sec^2\theta\, d\theta$

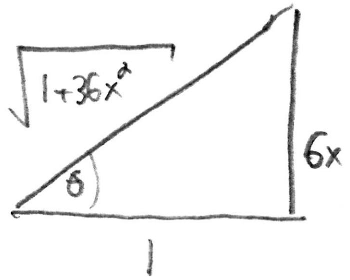
$= \int \frac{8}{6} \frac{1}{(\sec^2\theta)^{3/2}} \sec^2\theta\, d\theta$

$= \int \frac{4}{3} \frac{1}{\sec^3\theta} \sec^2\theta\, d\theta$

$= \int \frac{4}{3}\cos\theta\, d\theta$

$= \frac{4}{3}\sin\theta + C$

Draw a triangle



$6x = \tan\theta \to \tan\theta = \frac{6x}{1}$

So $I = \frac{4}{3}\,\frac{6x}{\sqrt{1+36x^2}} + C = \boxed{\frac{8x}{\sqrt{1+36x^2}} + C}$

37. $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{\cancel{4}^{2}\sin^2\theta\sqrt{4-4\sin^2\theta}}\,\cancel{2}\cos\theta\,d\theta$

$a^2 = 4 \to a = 2$

$u^2 = x^2 \to u = x$

"−" is in front of a^2

$\Rightarrow$ $u = a\sin\theta$

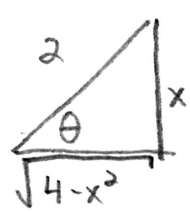
$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$= \int \frac{1}{\cancel{2}}\,\frac{1}{\sin^2\theta\cdot 2\cancel{\cos\theta}}\,\cancel{\cos\theta}$

$= \frac{1}{4}\int \csc^2\theta\,d\theta$

$I = -\frac{1}{4}\cot\theta + C.$



$x = 2\sin\theta \to \sin\theta = \frac{x}{2}$

$\Rightarrow \boxed{I = -\frac{1}{4}\,\frac{\sqrt{4-x^2}}{x} + C}$

38. $\int \frac{6}{100x^2-64}\,dx$

$u^2 = 100x^2 \to u = 10x$

$a^2 = 64 \to a = 8.$

"−" is in front of a

$\Rightarrow$ $u = a\sec\theta$

$10x = 8\sec\theta$

$x = \frac{4}{5}\sec\theta.$ $dx = \frac{4}{5}\sec\theta\tan\theta\,d\theta$

So $I = \int \frac{6}{100(\frac{16}{25}\sec^2\theta) - 64}\cdot\frac{4}{5}\sec\theta\tan\theta\,d\theta$

$= \int \frac{\cancel{6}^{3}}{\cancel{64}_{16\ 8}\tan^{\cancel{2}}\theta}\cdot\frac{\cancel{4}^{1}}{5}\sec\theta\cancel{\tan\theta}\,d\theta$

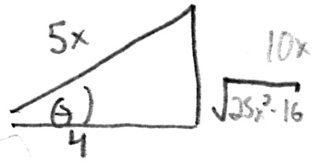
$$= \int \frac{3}{40} \frac{\sec\theta}{\tan\theta} \, d\theta \leftarrow$$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \frac{1}{\tan\theta} = \cot\theta = \frac{\cos\theta}{\sin\theta}$

so: $\frac{\sec\theta}{\tan\theta} = \frac{1}{\cos\theta} \frac{\cos\theta}{\sin\theta} = \csc\theta$

$$= \int \frac{3}{40} \csc\theta \, d\theta$$

$$= -\frac{3}{40} \ln|\csc\theta + \cot\theta| + C$$



$10x = 8\sec\theta \rightarrow \sec\theta = \frac{5x}{4}$

$$= \boxed{-\frac{3}{40} \ln\left|\frac{5x}{\sqrt{25x^2-16}} + \frac{4}{\sqrt{25x^2-16}}\right| + C}$$

39. a) We should use a u-substitution! The derivative of the expression under the radical is in the numerator

$u = 9x^2 - 1$

$du = 18x \, dx \quad \Rightarrow$

$\frac{du}{18x} = dx$

$$\int \frac{2x}{(9x^2-1)^{1/2}} dx = \int \frac{2\cancel{x}}{u^{1/2}} \frac{du}{18\cancel{x}}$$

$$= \frac{1}{9} \int u^{-1/2} du$$

$$= \frac{1}{9} \cdot 2u^{1/2} + C$$

$$= \boxed{\frac{2}{9}\sqrt{9x^2-1} + C}$$

b) With a trig substitution:

$u^2 = 9x^2 \rightarrow u = 3x$

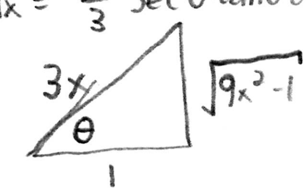
$a^2 = 1 \rightarrow a = 1$

"$-$" in front of $a^2 \Rightarrow$ use $u = a\sec\theta$

$3x = \sec\theta$

$x = \frac{1}{3}\sec\theta$

$dx = \frac{1}{3}\sec\theta\tan\theta \, d\theta$



$$\rightarrow I = \int \frac{\frac{2}{3}\sec\theta}{\sqrt{\cancel{9}(\frac{1}{\cancel{9}}\sec^2\theta)-1}} \frac{1}{3}\sec\theta\tan\theta \, d\theta$$

$$= \frac{2}{9} \int \frac{\sec\theta}{\cancel{\tan\theta}} \sec\theta \cancel{\tan\theta} \, d\theta$$

$$= \frac{2}{9} \tan\theta + C$$

$$= \boxed{\frac{2}{9}\sqrt{9x^2-1} + C}$$

They match!