Worksheet # 7:

I. Geometric Series

Determine if the following series converge or diverge. If they converge, find their value.

1. $\sum_{n=1}^{\infty} \frac{4}{3^n}$

2. $\sum_{n=3}^{\infty} 3^{2-3n}$

3. $\sum_{n=5}^{\infty} \frac{2^{3n+1}}{7^{n+100}}$

4. $\sum_{n=0}^{\infty} 4r^{2n}$

5. $\sum_{n=0}^{\infty} \frac{3^{2n+1}}{10^{n+2}}$

6. $\sum_{n=0}^{\infty} \frac{(-5)^n}{6^{2n}}$

7. $\sum_{n=0}^{\infty} 4(-1.75)^{\frac{n}{3}}$

8. $\sum_{n=1}^{\infty} \frac{2^{3n}}{60000}$

II. Divergence and Ratio Tests

Determine if the following series converge.

9. $\sum_{n=3}^{\infty} \frac{2-4n}{6+n}$

10. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

11. $\sum_{n=2}^{\infty} \frac{(n^2+1)\,3^{2n}}{(2n)!}$

12. $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$

13. $\sum_{n=1}^{\infty} \frac{6n^2-1}{\sqrt{n^4+7}}$

14. $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{2n}$

15. $\sum_{n=1}^{\infty} \cos\left(1-\frac{1}{n}\right)$

16. $\sum_{n=2}^{\infty} \frac{n^n}{n!}$

III Miscellaneous.

17. For which values of p does $\sum_{n=1}^{\infty} \frac{(pn)^n}{n!}$ converge?

18. Suppose $\sum_{n=1}^{N} a_n = \frac{4N-1}{3N^2}$.

a) What is $\sum_{n=1}^{\infty} a_n$?

b) What is $\lim_{n\to\infty} a_n$?

19. Consider $\sum_{n=1}^{\infty} a_n$. Suppose we are given $s_N = \sum_{n=1}^{N} a_n = \frac{4+N}{3-2N}$.

a) What is $a_1 + a_2 + a_3$?

b) What is $a_5 + a_6$?

c) What is $\sum_{n=1}^{\infty} a_n$?

d) What is $\lim_{n\to\infty} a_n$?

20. (Partial Sums). Write out s_1, s_2, s_3, s_4 for the following:

a) $\sum_{n=1}^{\infty} \frac{2n-1}{n^2}$

c) $\sum_{n=1}^{\infty} \frac{n}{2^n}$

b) $\sum_{n=1}^{\infty} \sin n\pi$

d) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$.

21. Explain how the sequence of partial sums, $\{s_N\}_{N=1}^{\infty}$ relates to finding whether $\sum_{n=1}^{\infty} a_n$ converges or diverges

IV Conceptual Questions

22. (True or False) Given $\sum_{n=1}^{\infty} a_n$ converges to $L>0$, determine if the following are true or false or cannot be determined.

a) $\lim_{n\to\infty} a_n = L$

b) $\lim_{n\to\infty} a_n = 0$

c) $\lim_{n\to\infty} s_n = L$

d) $\lim_{n\to\infty} s_n = 0$

e) $\sum_{n=1}^{\infty} s_n$ may converge

f) $\sum_{n=8}^{\infty} a_n$ converges to L

g) $\sum_{n=1}^{\infty} (a_n+1) = L+1.$

h) $\lim_{n\to\infty} (s_n - s_{n-1}) = L$

23. (True or False) Given $\sum_{n=1}^{\infty} a_n$ diverges, determine if the following are true or false or cannot be determined.

a) $\lim_{n\to\infty} a_n = \infty$

b) $\lim_{n\to\infty} s_n = \infty$

c) $\lim_{n\to\infty} a_n \neq 0.$

d) $\lim_{n\to\infty} (s_n - s_{n+1})$ DNE.

e) $\sum_{n=1}^{\infty} (a_{n+1} - a_n)$ may converge.

f) $\sum_{n=1}^{\infty} \frac{2}{a_n}$ may converge.

g) $\sum_{n=1}^{\infty} (a_n+1)$ must diverge

h) $\sum_{n=1}^{\infty} s_n$ must diverge

24. (True or False)

a) If $\sum_{n=1}^{\infty} a_n$ diverges, $\sum_{n=1000}^{\infty} a_n$ must diverge.

b) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

c) If $\sum_{n=1}^{\infty} a_n$ converges to L and $b_n = 5$ for all n,

$$\sum_{n=1}^{\infty} (a_n + b_n) = L + 5.$$

d) If $\sum_{n=1}^{\infty} a_n$ converges to L and $\lim_{n\to\infty} b_n = 5$, then

$$\sum_{n=1}^{\infty} (a_n + b_n) = L + 5.$$

e) If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$, then $\sum_{n=1}^{\infty} a_n$ must diverge.

f) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$, then $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ must converge.