Worksheet #9 : Parametric Equations and Polar Coordinates.

I. Parametric Equations

1. Eliminate the parameter and find a Cartesian representation of the given curves. Then, make a sketch and indicate the positive orientation.

   a) $x = 2t^2$, $y = 3t$ c) $x = 4e^t$, $y = e^{3t}$

   b) $x = 4\sin t + 1$, $y = 4\cos t$ d) $x = 2\cos t$, $y = \sin t$.

2. Give a parameterization of the circle $x^2 + (y-1)^2 = 4$. that:
   a) Starts at $(0,3)$ when $t=0$.
   b) Is traced out clockwise
   c) Starts at $(2,1)$ and is traced out twice between $t=0$ and $t=1$.

3. Suppose $x = at$ and $y = bt$.
   a) Show that this curve is the line $y = \frac{b}{a}x$.
   b) Find another parameterization of this curve that starts at $y = b$ when $t=0$.

4. Suppose $x = t^3 - 3t$, $y = t^2$.
   a) Find $\frac{dy}{dx}$ in terms of t.
   b) Find all horizontal and vertical tangent lines.
   c) Find the tangent line at $t=2$.
   d) Find the tangent line at $(x,y) = (2,1)$.

5. Suppose $x = t^2 - 6t + 1$, $y = \frac{1}{2}t^2 - t$
   a) Find $\frac{dy}{dx}$ in terms of t.
   b) Find all vertical and horizontal tangent lines.

c) Find the tangent line at $t=1$.

d) Find the tangent line at $(x,y) = (1, 12)$.

II. Polar Coordinates

6. Express the point $(x,y) = (0,1)$ in polar coordinates in 4 different ways.
7. Express the point $(x,y) = (-\sqrt{2}, -\sqrt{2})$ in polar coordinates in 4 different ways.
8. Express the point $(r,\theta) = (4, \frac{\pi}{3})$ in Cartesian coordinates.
9. Given the following curves $r=f(\theta)$, express the curves in Cartesian coordinates.

   a) $r = 4\sec\theta$ c) $1 = \sin\theta\cos\theta$

   b) $r^2 = \tan\theta$ d) $r = 5$

10. Let $r = 2\cos\theta$

    a) Find $\frac{dy}{dx}$ in terms of θ

    b) Find the vertical and horizontal tangent lines in Cartesian coordinates. Indicate the points of tangency in Cartesian Coordinates.

    c) Find the tangent line when $\theta = \frac{\pi}{6}$.

11. Repeat 10a), b) for $r = 2 + 2\sin\theta$
12. Find the area inside the curve $r = \sqrt{\cos\theta}$ and inside the circle $r = \frac{1}{\sqrt{2}}$.
13. Find the area of the region inside $r = 4\cos 2\theta$ and outside $r=2$.
14. Set up an integral that represents the area inside $r^2 = 2\sin 2\theta$ and outside $r=1$.
15. Set up an integral that represents the area between $r = 1+\sin\theta$ and $r = 1+\cos\theta$ in QI.