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General Information:

This midterm is a *sample* midterm. This means:

- The sample midterm contains problems that are of similar, but not identical, difficulty to those that will be asked on the actual midterm.
- The format of this exam will be similar, but not identical to the actual midterm. Note that this may be a departure from the format used on exams in previous semesters!
- The sample midterm is of similar length to the actual exam.
- Note that there are concepts covered this semester that do *NOT* appear on this midterm. This does not mean that these concepts will not appear on the actual exam! Remember, this midterm is only a sample of what *could* be asked, not what *will* be asked!

How to take the sample exam:

The sample midterm should be treated like the actual exam. This means:

- "Practice like you play." Schedule 55 minutes to take the sample exam and write answers as you would on the real exam; include appropriate justification, calculation, and notation!
- Do not refer to your books, notes, worksheets, or any other resources.
- You should not need (and thus, should not use) a calculator or other technology to help answer any questions that follow.
- The problems on this exam are mostly based on the Worksheets posted on the Math 1172 website and your previous quizzes.

  However, in your future professions, you will need to use mathematics to solve many different types of problems. As such, part of the goal of this course is:

  - to develop your ability to understand the broader mathematical concepts (not to encourage you just to memorize formulas and procedures!)
  - to apply mathematical tools in unfamiliar situations (Indeed, tools are only useful if you know when to use them!)

  There have been questions in your online homework and take-home quizzes with this intent, and there could be a problem on the exam that requires you to apply the material in an unfamiliar setting. To aid in preparation, there is such a problem on this sample exam.

How to use the solutions:

- Work each of the problems on this exam *before* you look at the solutions!
- *After* you have worked the exam, check your work against the solutions. If you are miss a type of question on this midterm, practice other types of problems like it on the worksheets!
- If there is a step in the solutions that you cannot understand, please talk to your TA or lecturer!

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Math 1172 Name: ______________________

Sample Midterm 3 OSU Username (name.nn): ______________________

Spring 2016 Lecturer: ______________________

Recitation Instructor: ______________________

Form A Recitation Time: ______________________

Instructions

- You have **55 minutes** to complete this exam. It consists of 6 problems on 8 pages including this cover sheet.
- If you wish to have any work on the extra workspace pages considered for credit, indicate in the problem that there is additional work on the extra workspace pages and **clearly label** to which problem the work belongs on the extra pages.
- The value for each question is both listed below and indicated in each problem.
- Please **write clearly** and make sure to **justify your answers** and **show all work**! Correct answers with no supporting work may receive no credit.
- You may not use any books or notes during this exam
- Calculators are NOT permitted. In addition, neither PDAs, laptops, nor cell phones are permitted.
- Make sure to read each question carefully.
- Please **CIRCLE** your final answers in each problem.
- A random sample of graded exams will be copied prior to being returned.

Problem	Point Value	Score
1	12	
2	18	
3	15	
4	20	
5	15	
6	10	
Total	100	

1. **Multiple Choice** [12 pts]

   Circle the response that best answers each question. Each question is worth 4 points. There is no penalty for guessing and no partial credit.

   I. The polar form of a curve in the xy plane is given by $r = \cos(2\theta)$.

   Which of the following is the Cartesian description of the curve?

   A. $x^2 + y^2 = x$

   B. $x^2 + y^2 = 2x$

   C. $(x^2 + y^2)^3 = (x^2 - y^2)^2$

   D. None of the above

   II. A curve is described parametrically by:

$$\begin{cases} x(t) & = 2\cos t \\ y(t) & = 3\sin t \end{cases}, \quad -\infty \leq t \leq \infty$$

   How many *distinct* vertical tangent lines does the curve have?

   A. 0

   B. 1

   C. 2

   D. More than 2

   III. Suppose that $\vec{u}$ is a nonzero vector, and $\hat{u}$ is a unit vector in the direction of $\vec{u}$. Then,

   A. $proj_{\vec{u}}\,\vec{u} = 0$.

   B. $proj_{\vec{u}}\,\vec{u} = \vec{u}$

   C. $proj_{\vec{u}}\,\vec{u} = \hat{u}$

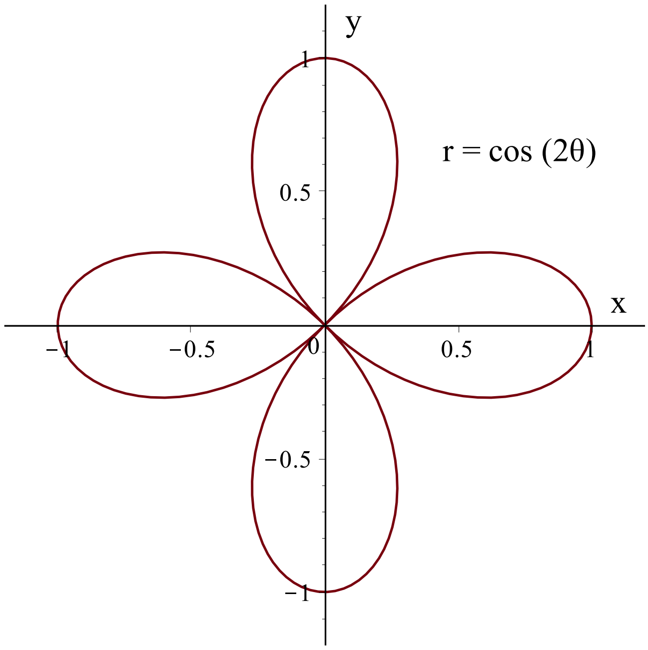
   D. It depends on what the vector $\vec{u}$ is.

2. **Multiselect** [18 pts]

Directions: Each problem below is worth 9 points. Circle *all* of the responses that MUST be true for each problem below. Note that there may be more than one correct response or even no correct responses!

A perfect answer for each part is worth 9 points. If you circle an incorrect choice, you will be penalized 3 points. If you do not circle a correct choice, you will be penalized 3 points. However, you cannot score below a 0 for either part of this problem. Thus, the possible scores for each part are 0, 3, 6, or 9 points.

I. The polar form of a curve in the xy plane is given by $r = \cos^2(\theta)$, which is shown below:



CIRCLE *all* of the following statements that MUST be true.

A. $\dfrac{dy}{dx} \geq 0$ for all θ in $[0, 2\pi]$.

B. $\dfrac{dy}{dx} \geq 0$ at each point where $x = \dfrac{1}{4}$.

C. $(x, y) = (0, 0)$ is on the curve.

D. There is an angle θ where $\dfrac{dy}{dx} = 6$.

E. There are 4 points on the curve where there are horizontal tangent lines.

F. There are 4 points on the curve where there are vertical tangent lines.

II. [9 pts] Suppose that $\vec{u}$ and $\vec{v}$ are nonzero three dimensional vectors.

CIRCLE *all* of the following statements that *MUST* be true.

A. If $|\vec{u}| = |\vec{v}|$, then $\vec{u} = \vec{v}$.

B. $proj_{\vec{u}}\,\vec{v} = proj_{\vec{v}}\,\vec{u}$.

C. If $\vec{u}$ and $\vec{v}$ are parallel, then $\vec{u} \cdot \vec{v} = 0$.

D. $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$

E. If $\vec{u}$ and $\vec{v}$ are orthogonal, then $\vec{u} \times \vec{v} = 0$.

F. $\vec{v} \cdot (\vec{u} - proj_{\vec{v}}\,\vec{u}\,) = 0$

3. [15 pts] Suppose that we define a new set of coordinates (u, v) by requiring that the usual Cartesian coordinates (x, y) are related to (u, v) as follows:

$$\begin{cases} x & = uv \\ y & = u^2 + v^2 \end{cases}$$

Now, suppose that a curve C is defined using these coordinates by the equation $u = v^2$.

I. Find the Cartesian coordinates (x, y) of the point on the curve when $v = 2$.

II. Find $\dfrac{dy}{dx}$ when $v = 2$.

III. Find the Cartesian description of the tangent line to the curve when $v = 2$.

Express your final answer in the form $y = mx + b$.

4. [20 pts] Suppose $\vec{r}(t) = \langle 2t^3,\, t^3 - 2,\, 1 - 2t^3 \rangle, \quad 0 \le t \le a.$

   I. Find the unit tangent vector $\hat{T}(t)$ for this curve.

   II. Find a value for a so the length of this curve is 24.

III. Explain why the curve is not parameterized by arclength. Then, find another description of the curve that uses arclength as a parameter.

Don't forget to give the domain of the arclength parameterization!

IV. Find the parametric equations of the tangent vector to this curve at $t = 1$.

5. [10 pts] Suppose $f(x,y) = \dfrac{2x^4 - x^2y^2}{x^2y^2 + 2y^4}$.

Determine whether $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists or does not exist. Explain your answer!

6. [15 pts]

   I. Find the equation of the plane that passes through the points $(2, -1, 0)$, $(1, 2, -1)$, and $(0, 1, 0)$.

   II. Show that the curve $\vec{r}(t) = \langle t^2 + 3, 2t^3 - t^2 - 2, -t^3 \rangle$ lies on the plane $x + y + 2z = 1$.

Answers:

1. **Multiple choice**

   I. C.

   II. C.

   III. B.

2. **Multiselect** The choices below should be circled!

   I. C, D

   II. D, F.

3. I. $(x, y) = (8, 20)$

   II. $\left.\dfrac{dy}{dx}\right|_{v=2} = 3$

   III. $y = 3x - 4$

4. I. $\hat{T}(t) = \left\langle \dfrac{2}{3}, \dfrac{1}{3}, \dfrac{2}{3} \right\rangle$

   II. $a = 2$

   III. The curve is not parameterized by arclength since $|\vec{r}\,'(t)| \neq 1$ for all t.

   An arclength parameterization for the curve is
   $\vec{r}\,(s) = \left\langle \dfrac{2s}{3}, \dfrac{s}{3} - 2, 1 - \dfrac{2s}{3} \right\rangle, 0 \leq s \leq 24.$

   IV. The tangent line is given by the vector-valued function $< 6t + 2, 3t - 1, -6t - 1 >$.

5. The limit does not exist; see solutions for the explanations

6. I. $x + y + 2z = 1$

   II. (see solutions)