INSTRUCTIONS

● SHOW ALL WORK in problems 1, 2, and 5.
   Incorrect answers with work shown may receive partial credit, but unsubstantiated correct answers may receive NO credit.

You don’t have to show work in problems 3 and 4.

● Give EXACT answers unless asked to do otherwise.

● Calculators are NOT permitted!
   PDA’s, laptops, and cell phones are prohibited.
   Do not have these devices out!

● The exam duration is 55 minutes.

● The exam consists of 5 problems starting on page 2 and ending on page 8. Make sure your exam is not missing any pages before you start.

<table>
<thead>
<tr>
<th>PROBLEM NUMBER</th>
<th>SCORE</th>
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<tbody>
<tr>
<td>1</td>
<td>(16)</td>
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<td>2</td>
<td>(18)</td>
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<td>3</td>
<td>(18)</td>
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<td>4</td>
<td>(30)</td>
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<td>5</td>
<td>(18)</td>
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<td>TOTAL</td>
<td>(100)</td>
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1. (16 pts)
   (a) (6 pts) Fill in the blanks.

   \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
   if the limit exists.

   (b) (10 pts) Let \( f(x) = \frac{1}{x+4} \).

   Use the (limit) definition of derivative in (a) to find \( f'(x) \).

   DO NOT USE THE PRODUCT OR QUOTIENT RULE! SHOW YOUR WORK!
2. (18 pts) A part of the curve with equation \( \cos(\pi xy) + x + y = 1 \) is sketched below.

(a) Use implicit differentiation to find the derivative \( \frac{dy}{dx} \).

(b) Consider the point \((1, 1)\). Show (algebraically) that this point lies on the curve.

(c) Find the equation of the line tangent to the curve at \((1, 1)\). Draw this line in the figure above.
MIDTERM 2
Form A, Page 4

3. (18 pts) MULTIPLE CHOICE!!!

A table of values for \( f(x) \) and \( f'(x) \) is shown below. Suppose that \( f \) is a one-to-one function and \( f^{-1}(x) \) is its inverse.

<table>
<thead>
<tr>
<th>x</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

CIRCLE THE CORRECT ANSWER IN EACH PART.

(I) Evaluate \( f^{-1}(f(x)) \) at \( x = 3 \).

(a) 1; (b) 3; (c) 6; (d) 4;

(e) DOES NOT EXIST; (f) NONE OF THE PREVIOUS ANSWERS.

(II) Evaluate \( \frac{d}{dx} f(f(x)) \) at \( x = 3 \).

(a) 6; (b) 25; (c) 5; (d) 15;

(e) DOES NOT EXIST; (f) NONE OF THE PREVIOUS ANSWERS.

(III) Evaluate \( \frac{d}{dx} \ln(f(x)) \) at \( x = 3 \).

(a) \( \frac{1}{4} \); (b) 5; (c) \( \frac{5}{4} \); (d) \( \frac{1}{5} \);

(e) DOES NOT EXIST; (f) NONE OF THE PREVIOUS ANSWERS.
3. (CONTINUED)

(IV) Evaluate $f^{-1}(x)$ at $x = 3$.

(a) $4$ ;
(b) $1$ ;
(c) $\frac{1}{3}$ ;
(d) $5$ ;

(e) DOES NOT EXIST ;
(f) NONE OF THE PREVIOUS ANSWERS.

(V) Evaluate $\frac{d}{dx}f^{-1}(x)$ at $x = 3$.

(a) $1$ ;
(b) $4$ ;
(c) $\frac{1}{5}$ ;
(d) $\frac{1}{4}$ ;

(e) $5$ ;
(f) NONE OF THE PREVIOUS ANSWERS.

(VI) Find the average rate of change of $f$ over the interval $[1, 3]$.

(a) $2$ ;
(b) $1$ ;
(c) $\frac{1}{2}$ ;
(d) $5$ ;

(e) DOES NOT EXIST ;
(f) NONE OF THE PREVIOUS ANSWERS.
4. (30 pts) EXPLANATION IS NOT REQUIRED, AND NO PARTIAL CREDIT WILL BE GIVEN.

(I) The (entire) graph of a function $f$ is shown in the figure below.

![Graph of function $f$]

(a) Find the $x$-coordinates of all critical points of $f$ (or write NONE).

**ANSWER:** critical point(s) at $x =$

(b) Find the $x$-coordinates of all local minima of $f$ (or write NONE).

**ANSWER:** local min(s) at $x =$

(c) Find all values of $x$ at which $f$ attains its global minimum (or write NONE).

**ANSWER:** global min(s) at $x =$

(d) Find the interval (or intervals) on which the derivative of $f$ is increasing.

**ANSWER:** derivative of $f$ is increasing on

(II) The figure below shows the graphs of $f$, $f'$, and $f''$.

![Graph of functions $f$, $f'$, and $f''$]

Which curve is which?

Fill in the blanks: $A =$ __; $B =$ __; $C =$ __.
4. EXPLANATION IS NOT REQUIRED, AND NO PARTIAL CREDIT WILL BE GIVEN.

(III) A function \( f' \) (derivative of \( f \)) is given 
\[
 f' (x) = (x - 2) e^x.
 \]

The following questions are about the function \( f \).

(a) Find all critical points of \( f \) (or write NONE).

\[
\text{ANSWER: critical point (s) at } x =
\]

(b) On what interval (or intervals) is \( f \) increasing?

\[
\text{ANSWER: } f \text{ is increasing on }
\]

(c) Find the point or points where \( f \) has a local maximum (or write NONE).

\[
\text{ANSWER: local max at } x =
\]

(d) Find the point or points where \( f \) has a local minimum.

\[
\text{ANSWER: local min (s) at } x =
\]

(e) Find \( f''(x) \), the second derivative of \( f \).

\[
\text{ANSWER: } f''(x) =
\]

(f) Identify any inflection points.

\[
\text{ANSWER: inflection point (s) at } x =
\]

(g) Determine the intervals on which the function is concave up or concave down.

\[
\text{ANSWER: } f \text{ is CONCAVE UP on }
\]
\[
\text{f is CONCAVE DOWN on}
\]
5. (18 pts) A right cone has fixed slant height (see figure) of 9 ft. The cone's height is shrinking at a rate of 0.5 ft/sec. At what rate is the area of the base changing when the height is 6 ft?

Make sure to label the picture.