INSTRUCTIONS

- **SHOW ALL WORK** in problems 1 and 4.
  Incorrect answers with work shown may receive partial credit,
  but unsubstantiated correct answers may receive NO credit.

  You don’t have to show work in problems 2 and 3.

- Give **EXACT answers** unless asked to do otherwise.

- Calculators are **NOT permitted**!
  PDA’s, laptops, and cell phones are prohibited.
  Do not have these devices out!

- The exam duration is 55 minutes.

- The exam consists of 4 problems starting on page 2 and ending on page 8.
  Make sure your exam is not missing any pages before you start.

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<th>PROBLEM NUMBER</th>
<th>SCORE</th>
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<td>1</td>
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<td>2</td>
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<td><strong>TOTAL</strong></td>
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1. (46 pts) SHOW YOUR WORK!

(I) (10 pts) Evaluate the limit. You may use L’Hospital’s Rule.

\[
\lim_{x \to 0} \left( \frac{\cos x - 1 + x^2}{x^2} \right)
\]

(II) (12 pts)

The **limit of Riemann sums** for a function \( f \) on the interval \([1, 5]\) is given by

\[
\lim_{\Delta \to 0} \sum_{k=1}^{n} \left( 2 \frac{x_k^*}{x_k} + \frac{1}{x_k} \right) \Delta x_k \quad \text{on} \ [1, 5].
\]

(a) Identify \( f \) and express the limit as a **definite integral**.

(b) Evaluate the **limit of Riemann sums**.
1. (46 pts) SHOW YOUR WORK!

(III) (12 pts)
Given the acceleration function of an object moving along a line, find the position function with the given initial velocity and position.

\[ a(t) = 2t, \quad v(0) = 2, \quad s(0) = 0 \]

(IV) (12 pts)

(a) Find the linearization, \( L(x) \), of the function

\[ f(x) = e^{2x} \] at \( a = 0 \).

(b) Using the linearization, \( L(x) \), from the part (a), approximate \( e \).
2. (18 pts) MULTIPLE CHOICE!

The figure shows the graph of a function $f$. Let $L_a(x)$ be the linear approximation of $f$ at $a$.

Circle ALL the correct statements below.

(a) $L_a(b) < f(b)$; (b) $L_a(b) > f(b)$; (c) $L_a(a) < f(a)$; (d) $L_a(a) > f(a)$; (e) No statement (a) – (d) is correct.

Let $g$ be a continuous function on $[3, 5]$, such that $g(3) = 2$, $g(5) = 8$, and $g'(x) > 0$ for all $x$ in $(3, 5)$.

Circle ALL the correct statements below.

(a) $0 < g'(x) \leq 2$, for all $x$ in $(3, 5)$; (b) $0 < g(x) \leq 8$, for all $x$ in $(3, 5)$; (c) $g'(c) = 3$, for some number $c$ in $(3, 5)$; (d) $g$ does not satisfy the conditions of the Mean Value Theorem on $[3, 5]$; (e) No statement (a) – (d) is correct.
2. (18 pts) MULTIPLE CHOICE!

(III) Given the four functions on interval [1, 5], answer the questions below.

(A) \[
\begin{array}{c}
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\end{array}
\]

(B) \[
\begin{array}{c}
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\end{array}
\]

(C) \[
\begin{array}{c}
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\end{array}
\]

(D) \[
\begin{array}{c}
\text{\hspace{1cm}} \\
\text{\hspace{1cm}} \\
\end{array}
\]

(i) Circle the function that satisfies (or functions that satisfy) the conditions of the Mean Value Theorem on [1, 5].

A B C D

(ii) Circle the function (or functions) for which there exists a point c in (1, 5) such that \[ f'(c) = \frac{f(5) - f(1)}{5 - 1}. \]

A B C D
3. (18 pts) MULTIPLE CHOICE! CIRCLE THE CORRECT ANSWER IN EACH PART.

The graph of $g$, a continuous function on $[0, 4]$, is shown in the figure.

Let

$$A(x) = \int_0^x g(t) \, dt, \text{ for } 0 \leq x \leq 4.$$ 

(I) (3 pts) Circle the correct statement about $A(2)$.

(a) $A(2) = 0$;  
(b) $A(2) > 0$;  
(c) $A(2) < 0$;  
(e) NONE OF THE PREVIOUS ANSWERS.

(II) (3 pts) Circle the correct statement about $A(3.8)$.

(a) $A(3.8) = 0$;  
(b) $A(3.8) > 0$;  
(c) $A(3.8) < 0$;  
(e) NONE OF THE PREVIOUS ANSWERS.

(III) (3 pts) Circle the correct statement about $A'(3.8)$.

(a) $A'(3.8) = 0$;  
(b) $A'(3.8) > 0$;  
(c) $A'(3.8) < 0$;  
(e) NONE OF THE PREVIOUS ANSWERS.
3. (18 pts) MULTIPLE CHOICE! CIRCLE THE CORRECT ANSWER IN EACH PART.

(IV) (3 pts)

Find the solution of the following initial value problem:

\[ y' (x) = g(x); \quad y(0) = 2 \]

(a) \( y(x) = g(x); \) \quad (b) \( y(x) = g(x) + 2; \)
(c) \( y(x) = A(x); \) \quad (d) \( y(x) = A(x) + 2; \)
(e) \( y(x) = g'(x); \) \quad (f) \( y(x) = g'(x) + 2; \)

(g) NONE OF THE PREVIOUS ANSWERS.

(V) (3 pts)

Find the expression for

\[ \int_{0}^{4} |g(t)| \, dt. \]

(a) \( A(4); \) \quad (b) \( A(2) - A(4); \) \quad (c) \( A(4) - A(2); \)
(d) \( A(4) - 2A(2); \) \quad (e) NONE OF THE PREVIOUS ANSWERS.

(VI) (3 pts)

Find the midpoint Riemann sum for the function \( g \)

on the interval \([0, 4]\)

with \( n = 2 \) (the number of subintervals).

(a) \( g(1) + g(3); \) \quad (b) \( g(0) + g(2); \) \quad (c) \( g(2) + g(4); \)
(d) \( 2g(1) + 2g(3); \) \quad (e) \( 2g(0) + 2g(2); \) \quad (f) \( 2g(2) + 2g(4); \)

(g) NONE OF THE PREVIOUS ANSWERS.
4. (18 pts) A part of a circle centered at the origin with radius $r = 7$ cm is given in the figure (A) below. A right triangle is formed in the first quadrant (see figure (A)). One of its sides lies on the $x$-axis. Its hypotenuse runs from the origin to a point on the circle. The hypotenuse makes an angle $\theta$ with the $x$-axis.

Make sure to label the picture.

(a) Draw 2 more examples of such a triangle in the figure (B).

(b) Express the area of such a triangle as a function of $\theta$ and state its domain.

A ($\theta$) = 

Domain of A = 

(c) Find the value of $\theta$ which maximizes the area in part (b). Show your work and justify your answer.