INSTRUCTIONS

• SHOW ALL WORK in problems 1, 2, and 5. Incorrect answers with work shown may receive partial credit, but unsubstantiated correct answers may receive NO credit.

You don’t have to show work in problems 3 and 4.

• Give EXACT answers unless asked to do otherwise.

• You do not need to simplify numerical answers such as \( \frac{5}{\sqrt{8}} - \frac{3}{\sqrt{32}} \).

• Calculators are permitted EXCEPT those calculators that have computer algebra systems (CAS) or ability to communicate with others. Furthermore, all memory must be cleared and all apps must be removed. PDA’s, laptops, and cell phones are prohibited. Do not have these devices out!

• The exam duration is 55 minutes.

• The exam consists of 5 problems starting on page 2 and ending on page 8. Make sure your exam is not missing any pages before you start.

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1. (20 pts)

A rectangle is constructed with its base on the x-axis and two of its vertices on the parabola \( y = 36 - x^2 \) and above the x-axis.

(I) Make a sketch and label it.

(II) What are the dimensions of the rectangle with the maximum area? Justify your answer!

(III) What is that maximum area?
2. (20 pts)

(I) (10 pts) Evaluate the limit. You may use L'Hospital's Rule.

\[ \lim_{{x \rightarrow 0^+}} (1 + 3x)^{\frac{4}{x}} \]

(II) (10 pts) Given the acceleration function of an object moving along a line, find the position function with the given initial velocity and position.

\[ a(t) = 4t, \quad v(0) = 3, \quad s(0) = 5 \]
3. (22 pts)

(I) (10 pts) EXPLANATION IS NOT REQUIRED, AND NO PARTIAL CREDIT WILL BE GIVEN.

The figure shows the graph of a function $f$ on the interval $[3, 7]$.

(a) Find the average rate of change of the function $f$ on the interval $[3, 7]$.

(b) Find the slope of the secant line that passes through $(3, 2)$ and $(7, 0)$.

(c) Sketch the secant line from the part (b) in the figure above.

(d) In the figure above mark the points $P$ (if they exist) at which the slope of the tangent line equals the slope of the secant line from the part (b).

(e) Name the theorem that guarantees that such a point $P$ exists.

(f) Sketch the tangent line at $P$ in the figure above.
3. (CONTINUED) MULTIPLE CHOICE! CIRCLE THE CORRECT ANSWER IN EACH PART.

(II) (3 pts)
Find the equation of the line that represents the linear approximation to the function \( f(x) = e^{2x} \) at \( a = 0 \).

(a) \( y = 1 + 2x \); (b) \( y = 1 - 2x \); (c) \( y = \ln(2x + 1) \); (d) \( y = x \);
(e) SUCH A LINE DOES NOT EXIST; (f) NONE OF THE PREVIOUS ANSWERS.

(III) (3 pts)
Determine the indefinite integral \( \int \left( \frac{1}{1 + x^2} - 3 \right) \, dx \).

(a) \( \ln(1 + x^2) + C \); (b) \( \ln(1 + x^2) - 3x + C \); (c) \( \tan^{-1}x - 3x + C \);
(d) \( \tan^{-1}x \); (e) \( \tan^{-1}x + C \); (f) NONE OF THE PREVIOUS ANSWERS.

(IV) (3 pts)
Evaluate the sum \( \sum_{k=1}^{3} (1 + k^2) \).

(a) 7; (b) 15; (c) 17; (d) \( \frac{k(k + 1)(2k + 1)}{6} \);
(e) 16; (f) NONE OF THE PREVIOUS ANSWERS.

(V) (3 pts)
Given that \( \int_{0}^{6} g(x) \, dx = 2 \), evaluate the integral \( \int_{0}^{6} (4g(x) + 1) \, dx \).

(a) 2; (b) 8; (c) 26; (d) 54; (e) 14; (f) NONE OF THE PREVIOUS ANSWERS.
4. (18 pts) MULTIPLE CHOICE! CIRCLE THE CORRECT ANSWER IN EACH PART.

The graph of \( f \) is shown in the figure. Let

\[
A(x) = \int_{0}^{x} f(t) \, dt, \text{ for } 0 \leq x \leq 6, \quad \text{and} \quad F(x) = \int_{3}^{x} f(t) \, dt, \text{ for } 3 \leq x \leq 6.
\]

be two area functions for \( f \).

(I) (3 pts) Evaluate \( A(3) \).

(a) \(-2\); (b) \(2\); (c) \(1\); (d) \(-1\); (e) \(0\);

(f) \(-\frac{3}{2}\); (g) \(\frac{3}{2}\) (h) NONE OF THE PREVIOUS ANSWERS.

(II) (3 pts) Evaluate \( F(3) \).

(a) \(2\); (b) \(1\); (c) \(0\); (d) \(\frac{3}{2}\);

(e) \(\frac{1}{2}\) (f) NONE OF THE PREVIOUS ANSWERS.
4. (18 pts) MULTIPLE CHOICE! CIRCLE THE CORRECT ANSWER IN EACH PART.

(III) (3 pts) Evaluate $A'(1.5)$.

(a) 0; (b) $-1$; (c) 1; (d) $\frac{3}{2}$; (e) $\frac{1}{2}$; (f) NONE OF THE PREVIOUS ANSWERS.

(IV) (3 pts) Find an expression for $F'(x)$, for $3 \leq x \leq 6$.

(a) $F'(x) = 1$; (b) $F'(x) = 3 - x$; (c) $F'(x) = \frac{-(3 - x)^2}{2}$; (d) $F'(x) = -(3 - x)^2$; (e) NONE OF THE PREVIOUS ANSWERS.

(V) (3 pts) Find an expression for $F(x)$, for $3 \leq x \leq 6$.

(a) $F(x) = 1$; (b) $F(x) = 3 - x$; (c) $F(x) = \frac{-(3 - x)^2}{2}$; (d) $F(x) = -(3 - x)^2$; (e) NONE OF THE PREVIOUS ANSWERS.

(VI) (3 pts) Evaluate $\int_{0}^{5} |f(t)| \, dt$.

(a) 0; (b) 6; (c) 4; (d) 2; (e) 1; (f) NONE OF THE PREVIOUS ANSWERS.
5. (20 pts)

(a) Using the Fundamental theorem of calculus, evaluate the integral

\[ \int_0^6 (4 x - x^2) \, dx. \]

(b) Use the midpoint Riemann sum with \( n = 3 \) to approximate the value of

the integral \( \int_0^6 (4 x - x^2) \, dx. \)

(c) Illustrate the midpoint Riemann sum from the part (b) by sketching the appropriate rectangles in the figure below.

\[ y = f(x) = 4x - x^2 \]