Worksheet #9: Parametric Equations and Polar Coordinates.

I. Parametric Equations

1. Eliminate the parameter and find a Cartesian representation of the given curves. Then, make a sketch and indicate the positive orientation.
   
   a) \( x = 2t^2, \ y = 3t \)
   
   b) \( x = 4 \sin t + 1, \ y = 4 \cos t \)
   
   c) \( x = 4e^t, \ y = e^{2t} \)
   
   d) \( x = 2 \cos t, \ y = \sin t \)

2. Give a parameterization of the circle \( x^2 + (y-1)^2 = 4 \), that:
   
   a) Starts at \((0,3)\) when \(t=0\).
   
   b) Is traced out clockwise.
   
   c) Starts at \((2,1)\) and is traced out twice between \(t=0\) and \(t=1\).

3. Suppose \( x = at \) and \( y = bt \).
   
   a) Show that this curve is the line \( y = \frac{b}{a} x \).
   
   b) Find another parameterization of this curve that starts at \( y = b \) when \( t=0 \).

4. Suppose \( x = t^3 - 3t, \ y = t^2 \).
   
   a) Find \( \frac{dy}{dx} \) in terms of \( t \).
   
   b) Find all horizontal and vertical tangent lines.
   
   c) Find the tangent line at \( t = 2 \).
   
   d) Find the tangent line at \((x,y) = (2,1)\).

5. Suppose \( x = t^2 - 6t + 1, \ y = \frac{1}{2}t^3 - t \).
   
   a) Find \( \frac{dx}{dt} \) in terms of \( t \).
   
   b) Find all vertical and horizontal tangent lines.
c) Find the tangent line at \( t=1 \).
d) Find the tangent line at \((x, y) = (1, 12)\).

II. Polar Coordinates

6. Express the point \((x, y) = (0, 1)\) in polar coordinates in 4 different ways.

7. Express the point \((x, y) = (-\sqrt{2}, -\sqrt{2})\) in polar coordinates in 4 different ways.

8. Express the point \((r, \theta) = (4, \frac{\pi}{3})\) in Cartesian coordinates.

9. Given the following curves \( r=f(\theta) \), express the curves in Cartesian coordinates.
   a) \( r = 4 \sec \theta \)
   b) \( r^2 = \tan \theta \)
   c) \( l = \sin \theta \cos \theta \)
   d) \( r = 5 \)

10. Let \( r = 2 \cos \theta \)
   a) Find \( \frac{dy}{dx} \) in terms of \( \theta \)
   b) Find the vertical and horizontal tangent lines in Cartesian coordinates. Indicate the points of tangency in Cartesian coordinates.
   c) Find the tangent line when \( \theta = \frac{\pi}{6} \).

11. Repeat 10(a), b) for \( r = 2 + 2\sin \theta \)

12. Find the area inside the curve \( r = \sqrt{\cos \theta} \) and inside the circle \( r = \frac{1}{\sqrt{2}} \).

13. Find the area of the region inside \( r = 4 \cos 2\theta \) and outside \( r = 2 \).

14. Set up an integral that represents the area inside \( r^2 = 2 \sin 2\theta \) and outside \( r = 1 \).

15. Set up an integral that represents the area between \( r = 1 + \sin \theta \) and \( r = 1 + \cos \theta \) in \( QT \).