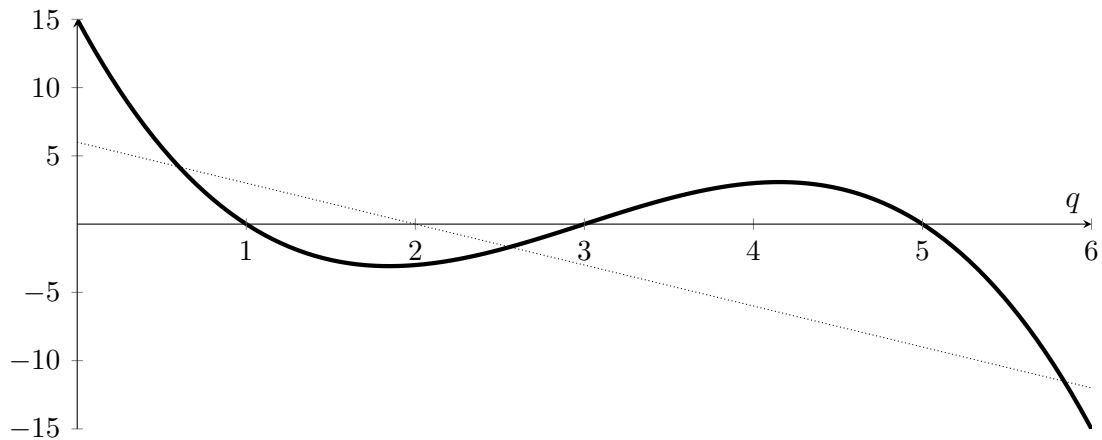


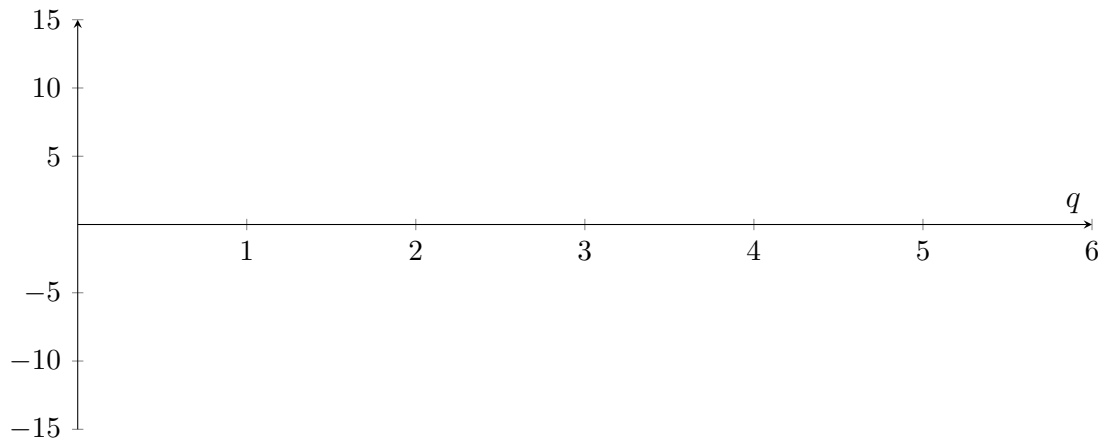
1. The graphs of the two functions $y = 6 - 3x$ and $y = -x^3 + 9x^2 - 23x + 15$ are below.



The piecewise-defined function $P(x)$ is given by:

$$P(x) = \begin{cases} 6 - 3x & \text{if } 0 \leq x < 3 \\ -x^3 + 9x^2 - 23x + 15 & \text{if } 3 \leq x \leq 6 \end{cases}$$

- (a) (10 points) Sketch the graph of $y = P(x)$.



- (b) (4 points) The range of $P(x)$ is _____ (use interval notation)

2. Answer each of the following short answer questions. You do **not** need to show your work.

- (a) (10 points) A 2019 Honda CR-V vehicle gets 28 miles per gallon driving on city streets and 34 miles per gallon driving on highways. The fuel tank for the CR-V holds 14 gallons. Suppose you drove 431 miles on a full fuel tank. Let x be the amount (in gallons) of fuel used for city driving and y be the amount (in gallons) used for highway driving. Set up the system of two equations equations that will solve for x and y . **DO NOT SOLVE THIS SYSTEM OF EQUATIONS.**

- (b) (6 points) Let $A = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -4 & 3 & 1 & 3 \\ -4 & 1 & 2 & 0 & -1 \end{pmatrix}$. Then the product AB is a $\underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$ matrix.

- (c) (6 points) The sequence below is geometric. Determine the missing term.

$$4, \underline{\hspace{1cm}}, 196, 1372$$

- (d) (8 points) The **demand** equation for a product is given by $p = 72 - 8q$. The **cost** equation for producing q units is $C = 6 + 8q$. Determine the **profit** function $P(q)$.

$$P(q) = \boxed{\hspace{10cm}}$$

3. Answer each of the following short answer questions. You do **not** need to show your work.

(a) (6 points) Solve the inequality below. Write your answer in interval notation.

$$\frac{3t + 1}{-3} < \frac{7 - 2t}{-3}$$

Answer:

(b) (6 points) Determine the composition $(f \circ g)(x)$ for $f(x) = e^x$ and $g(x) = x^2 + x$.

$$(f \circ g)(x) =$$

(c) (6 points) Give an equation of the line that passes through the point $(-4, 6)$ that is perpendicular to the line $y = 2x + 3$. Write the equation in either point-slope form or slope-intercept form.

Answer:

(d) (6 points) Determine A and B in the matrix equation $AX = B$, with A as the coefficient matrix of the following system of equations and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{cases} x - y + 3z = 4 \\ 2x - 3y = 0 \\ x + z = -2 \end{cases}$$

$$A = \begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix} \quad B = \begin{pmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{pmatrix}$$

4. Answer the following interest theory questions.

- (a) (10 points) Suppose \$4000 is deposited into an account that gains interest at a rate of 6% compounded continuously. At time t (in years), the account has \$6200. Find t , rounded to two decimal places. **Write the formula that you use to calculate t .**

$$t = \boxed{}$$

- (b) (10 points) Abe deposits \$500 into an investment fund at the end of every quarter. The fund gains interest at 3%, compounded quarterly. After 20 years, the account is worth X dollars. Find X , rounded to two decimal places. **Write the formula that you use to calculate X .**

$$X = \boxed{}$$

- (c) (10 points) You borrow \$30,000 which will be repaid over the next 5 years with monthly payments of Y . The interest rate charged on the loan is 5.4% compounded monthly. Find Y , rounded to two decimal places. **Write the formula that you use to calculate Y .**

$$Y = \boxed{}$$

5. Solve the equations. Separate multiple solutions with a comma. Show all of your work. **Solutions by calculator will receive no credit.**

(a) (12 points)

$$x + \sqrt{x + 5} = 7$$

$x =$

(b) (12 points)

$$\log_2(x - 5) + \log_2(x - 3) = 3$$

$x =$

(c) (12 points)

$$\log_x(20 - x) = 2$$

$x =$

6. (12 points) Solve the given nonlinear system of equations. Give your answers as ordered pairs of integers and/or fractions, and separate multiple solutions with commas. Show all of your work. **No credit will be given to calculator solutions.**

$$\begin{cases} x^2 + 3y^2 = 4 \\ x + 2y = -1 \end{cases}$$

$$(x, y) = \boxed{}$$

7. (12 points) You are given the following supply and demand equations:

$$\begin{cases} S(q) = \frac{q}{20} + 20 \\ D(q) = \frac{6000}{q} \end{cases}$$

Determine the equilibrium quantity, q . Show all of your work. **No credit will be given to calculator solutions.**

$$q = \boxed{}$$

8. (12 points) Let $f(x) = 3x^2 - 2x$. Determine the expression for $\frac{f(x+h) - f(x)}{h}$. You must simplify your result.

$$\frac{f(x+h) - f(x)}{h} =$$

9. (10 points) You are given that

$$\log x = -3 \quad \log y = 2 \quad \log z = 1$$

Compute

$$\log \left(\frac{x^3 \cdot y^{1.5}}{z} \right)$$

$$\log \left(\frac{x^3 \cdot y^{1.5}}{z} \right) =$$

10. Perform the indicated operations on the following matrices.

(a) (8 points) Let

$$A = \begin{bmatrix} 9 & -3 \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 8 \\ 0 & 1 \end{bmatrix}$$

Determine the product AB

$$AB = \begin{bmatrix} & \\ & \end{bmatrix}$$

(b) (12 points) Use an augmented matrix, $[C|I]$, and elementary row operations to find the inverse of the following matrix:

$$C = \begin{bmatrix} 5 & 1 \\ 11 & 2 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Scrap work

Some Useful Formulas

$$S = P(1 + r)^n$$

$$P = S(1 + r)^{-n}$$

$$r_e = \left(1 + \frac{r}{n}\right)^n - 1$$

$$S = Pe^{rt}$$

$$P = Se^{-rt}$$

$$r_e = e^r - 1$$

$$A = Ra_{\overline{n}|r} = R \frac{1 - (1 + r)^{-n}}{r}$$

$$R = \frac{A}{a_{\overline{n}|r}} = A \frac{r}{1 - (1 + r)^{-n}}$$

$$S = Rs_{\overline{n}|r} = R \frac{(1 + r)^n - 1}{r}$$

$$\sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1 - r}$$

$$\sum_{i=1}^k ar^{i-1} = \frac{a(1 - r^k)}{1 - r}$$

$$\text{Int}_k = R \cdot [1 - (1 + r)^{-n+k-1}]$$

$$\text{Prin}_k = R \cdot (1 + r)^{-n+k-1}$$